۱۹,۵

تکلیف شماره ۹   کهاز مدرسی   نام مهم دفتر A

① (۲)

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$$

$$fof(\frac{2}{3}) \rightarrow f(\frac{2}{3}) = \cot \frac{2\pi}{12} = \cot \frac{\pi}{6} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = ②$$

② الف) $g(x) = 2\cos^2 x$, $f(\frac{1}{x}) = \sqrt{\frac{2x-1}{x^2}}$ (۲)

$$fog(\frac{\pi}{4}) \Rightarrow g(\frac{\pi}{4}) = 2 \times (\frac{1}{2})^2 = \frac{1}{2}$$

$$f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \boxed{\frac{\sqrt{3}}{2}}$$

ب) $g(x) = \frac{x}{1-x}$, $f(x) = [x]$

$$fog(\sqrt{2}) \rightarrow$$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$$

$$f(\sqrt{2}+2) = [-\sqrt{2}-2] = \boxed{-4}$$

③ $f(x) = \sin x$, $g(x) = x\sqrt{1-x^2}$ مطلوب $gof(\frac{\pi}{4})$ ؟ (۱,۵)

$$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad , \quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \sqrt{\frac{2}{4}}$$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

④ (۲)

الف) $f \circ g(x) = \{(۴,۵), (۲,۵), (۴,۱۲), (۸,۱۲)\}$

ب) $g \cdot f(x) = \emptyset$

ج) $f \cdot f(x) = \emptyset$

د) $g \circ g(x) = \{(۴,۸), (۲,۴), (۴,۱۰)\}$

⑤ (۳)

$g = \{(۱,۲), (۳,۱), (a,۳), (۱,b)\}$

$f = \{(۲,۱), (۳,۲), (۴,۵), (۱,۷)\}$

$(۳,۲) \in f \circ g$

$(۴,۱) \in g \circ f$

$a = ۴$

$f \circ g = \{(۱,۷), (۳,۷), (a,۲), (b,۷)\}$

$g \cdot f = \{(۳,۲), (۴,۱)\}$

$b = ۵$

$(a,b) = (۴,۵)$

⑥ (۳)

$f(f(x)) = ۴x + ۳$ $\quad g \circ f(-۱) = ?$

$g(۳x+۲) = ۳x - ۲$

$a(ax+b) + b = a^۲x + ab + b = ۴x + ۳ \rightarrow a^۲ = ۴ \rightarrow a = \pm ۲$

$ab + b = ۳$

$cx + d \xrightarrow{x = ۳x+۲} c((۳x+۲)) + d = ۳x - ۲ = ۳cx + ۲c + d \rightarrow \frac{۹}{۲} + d = -۲$

$۲c = ۳ \rightarrow c = \frac{۳}{۲}$ $\quad d = \frac{-۴.۹}{۲} = \frac{-۱۳}{۲}$

$ab+b=3$, $a=\pm 2$

$a=2 \rightarrow 2b+b=3b=3 \rightarrow b=+1$

$a=-2 \rightarrow -2b+b=-b=3 \rightarrow b=-3$

$f(x)=2x+1$ و $-2x-3$

$g(x)=\frac{3}{2}x-\frac{13}{2}$

$f(-1)=-2+1=(-1)$ و $2-3=-1$

$g(-1)=\frac{-3}{2}-\frac{13}{2}=\frac{-16}{2}=(-8) \Rightarrow g\circ f(-1)=-8$

$f(x)=\sqrt{x+|x|}$, $g(x)=\frac{1}{x^2-4x}$ (۷)

(۲)

$D_{g\circ f}=?$

$x\in D_f \rightarrow x+|x|\geq 0 \rightarrow x>0$ یا $x=0$

$x<0$ یا $x-x=0$

$f\in D_g \rightarrow x^2-4x\neq 0$

$x(x-4)\neq 0$

$x\neq 0 \rightarrow \sqrt{x+|x|}\neq 0 \rightarrow x>0$

$x\neq 4 \rightarrow \sqrt{x+|x|}\neq 4 \rightarrow x+|x|\neq 16$

$x\neq 8$

$D_{g\circ f}=(0,+\infty)-\{8\}$

۱) $f(x)=\sqrt{1-x^2}$, $g(x)=\sqrt{x}$:

$D_{(f+g)\circ f}$ ؟

$f+g(x)=\sqrt{1-x^2}+\sqrt{x}$

$f+g\left(\sqrt{1-x^2}\right)=\sqrt{1-\left(\sqrt{1-x^2}\right)^2}+\sqrt{\sqrt{1-x^2}}$

$=\sqrt{1-1+x^2}+\sqrt[4]{1-x^2}=\sqrt{x^2}+\sqrt[4]{1-x^2}$

$x^2\geq 0 \qquad 1-x^2\geq 0 \rightarrow 1\geq x^2 \rightarrow 1\geq x\geq -1$

$D_{(f+g)\circ f}=[-1,1]$

۲) الف) $f\left(\frac{3x+1}{x-2}\right)=6x+5$

$\frac{3x+1}{x-2}=t \rightarrow x=\frac{2t+1}{t-3} \Rightarrow f(t)=6\times\frac{2t+1}{t-3}+5$

$=\frac{18t+6+5t-15}{t-3}=\frac{23t-11}{t-3}$ $\qquad f(x)=\frac{23x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}$

$\left(x+\frac{1}{x}\right)^3=x^3+3x^2\times\frac{1}{x}+3x\times\frac{1}{x^2}+\frac{1}{x^3}=$

$x^3+3x+\frac{3}{x}+\frac{1}{x^3}$

$f\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^3-\frac{3}{x}-3x=\left(x+\frac{1}{x}\right)^3-3\left(\frac{1}{x}+x\right)$

$f(t)=t^3-3t \rightarrow f(x)=x^3-3x$

$f(x) = x\sqrt{x} \rightarrow f(0) = 0$ ، ۲ ، ۱۰

$g(1) = 0 \quad , \quad g(2\sqrt{2}) = 0$

$\int$

$f(x) = 1 \rightarrow x = 1$

$f(x) = 2\sqrt{2} \rightarrow x = 2$ $\Big\}$ $2 - 1 \neq 1$