

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$$

$$f \circ f\left(\frac{\sqrt{2}}{2}\right) = f\left(f\left(\frac{\sqrt{2}}{2}\right)\right) = ?$$

$$f\left(\frac{\sqrt{2}}{2}\right) = \cot \frac{\pi \times \frac{\sqrt{2}}{2}}{4} = \cot \frac{\pi}{4} = \sqrt{2}$$

$$f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = 2$$

$$f \circ f\left(\frac{\sqrt{2}}{2}\right) = 2$$

الف) $f(g(\frac{\pi}{4})) = f \circ g(\frac{\pi}{4})$ $g(\frac{\pi}{4}) = 2 \cos^2 \frac{\pi}{4} = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{2(1/2) - 1}{(1/2)^2}} = \frac{\sqrt{3}}{2} \rightarrow f \circ g\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

ب) $f = f(g(\sqrt{2}))$ $g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{(1 + \sqrt{2})}{(1 + \sqrt{2})} = \frac{\sqrt{2} + 2}{1 - 2} = -(\sqrt{2} + 2)$

$$f(-\sqrt{2} - 2) = \left[\frac{-\sqrt{2} - 2}{-1 - \sqrt{2}} \right] = -4$$

$$f \circ g(\sqrt{2}) = -4$$

$$g \circ f\left(\frac{\pi}{4}\right) = g\left(f\left(\frac{\pi}{4}\right)\right)$$

$$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{1}{2}$$

$$g \circ f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

$$g(x) = \{(x, 4), (2, 4), (4, 8), (8, 16)\}$$

$$f(x) = \{(x, 2), (4, 2), (8, 12), (16, 12)\}$$

الف) $f \circ g(x) = f(g(x)) = \{(x, 2), (2, 2), (4, 12), (8, 12)\}$

ب) $g \circ f(x) = g(f(x)) = \emptyset$

ج) $f \circ f(x) = f(f(x)) = \emptyset$

د) $g \circ g(x) = g(g(x)) = \{(x, 8), (2, 4), (4, 16)\}$

$$f = \{(x, 1), (3, 2), (4, 2), (1, 7)\}$$

$$g = \{(1, 2), (3, 1), (2, 3), (6, 1)\}$$

$(x, y) \in f \circ g \Rightarrow f(g(x)) = y \rightarrow g(x) = 3 \rightarrow x = 1 = x$ $(a, b) \rightarrow (f, a)$

$(x, y) \in g \circ f \rightarrow g(f(x)) = 1 \rightarrow f(x) = 3 \rightarrow x = 4 = x$ $f(4) = 2 = b$

$$f = ax + b \rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b = 4x + 1$$

$$\rightarrow f(-1) = 2(-1) + 1 = -1$$

$$2(-1) - 3 = -5$$

$$g(2x + 3) = 2x - 2$$

$$2x + 3 = -1 \rightarrow x = -2 \rightarrow g(-1) = 2(-2) - 2 = -6$$

$$g \circ f(-1) = g(f(-1)) = -6$$

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \rightarrow D_{g \circ f} = \textcircled{1} \cap \textcircled{2} \cap \textcircled{3} \rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$$D_f \Rightarrow x + |x| \geq 0 \rightarrow \text{همواره برقرار} \rightarrow D_f = \mathbb{R} \textcircled{4}$$

$$D_g \rightarrow \mathbb{R} - \{\text{ریشه های مربع}\} \rightarrow \begin{matrix} x^2 - 4x \neq 0 \\ x(x-4) \end{matrix} \rightarrow \begin{matrix} x \neq 0 \\ x \neq 4 \end{matrix} \rightarrow D_g = \mathbb{R} - \{0, 4\}$$

$$D_{g \circ f} = \{x \in \mathbb{R}, \sqrt{x+|x|} \in \mathbb{R} - \{0, 4\}\} \rightarrow \begin{matrix} \sqrt{x+|x|} \neq 0 \rightarrow x+|x| \neq 0 \rightarrow |x| \neq -x \rightarrow x > 0 \\ \sqrt{x+|x|} \neq 4 \rightarrow x+|x| \neq 16 \end{matrix} \rightarrow \begin{matrix} x > 0 \\ x+x \neq 16 \rightarrow x \neq 8 \end{matrix} \rightarrow D_{g \circ f} = (0, 8) \cup (8, +\infty)$$

$$D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{f+g}\}$$

$$D_{f+g} = D_f \cap D_g$$

$$D_f \Rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \textcircled{1}$$

$$D_g \Rightarrow x \geq 0 \textcircled{2}$$

$$D_{f+g} = \textcircled{1} \cap \textcircled{2} = [0, 1]$$

$$D_{(f+g) \circ f} = \{x \in [-1, 1], \sqrt{1-x^2} \in [0, 1]\} \xrightarrow{\cap} D_{(f+g) \circ f} = [0, 1]$$

$$\text{الف)} \quad \frac{3x+1}{x-2} = t \rightarrow tx - 2t = 3x+1 \rightarrow 3x - tx = -2t - 1$$

$$x(3-t) = -(2t+1) \rightarrow x = \frac{-(2t+1)}{3-t} \rightarrow f(t) = \frac{-(2t+1)}{3-t} + 2 = \frac{-2t-1+6-2t}{3-t} = \frac{-4t+5}{3-t}$$

$$= \frac{-13t+11}{3-t} \rightarrow f(x) = \frac{-13x+11}{3-x}$$

$$\text{ب)} \quad x + \frac{1}{x} = t \rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} = x^3 + \frac{1}{x^3} + 3$$

$$\rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 = t^3 - 3t \rightarrow f(t) = t^3 - 3t \rightarrow \boxed{f(x) = x^3 - 3x}$$

$$g \circ f(x) = g(f(x))$$

از آنجایی که تابع g محدوده x ها را در تقاطع به طول $2\sqrt{2}$ قطع می کند

پس $g(1) = 0$ و $g(2\sqrt{2}) = 0$ پس $f(x)$ باید مساوی 1 و $2\sqrt{2}$ باشد

تا در تابع g محدوده x ها را قطع کند. پس باید در تابع $f(x)$ x ها را

که $2\sqrt{2}$ و 1 هستند را به 1 و $2\sqrt{2}$ تبدیل نماید. $g \circ f(x)$ محدوده x ها را قطع می کند به دست آید

$$g(1) = 0 \text{ و } g(2\sqrt{2}) = 0 \rightarrow f(x) = 1 \text{ و } f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow \boxed{x=2}$$

$$\rightarrow x\sqrt{x} = 1 \rightarrow \boxed{x=1}$$

$$2-1=1$$