یازدهم دختر A — تکلیف شماره ۹ — 19,25 — مهناز محمدرضایی

1 — (3)

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

$fof(\frac{2}{3}) = f(f(\frac{2}{3})) = ?$

$f(\frac{2}{3}) = \cot\frac{\pi\times\frac{2}{3}}{4} = \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = 2$

$fof(\frac{2}{3}) = 2$

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2 — (2)

الف) $f(g(\frac{\pi}{3})) = fog(\frac{\pi}{3})$ $\quad g(\frac{\pi}{3}) = 2\underbrace{\cos^2\frac{\pi}{3}}_{\frac{1}{4}} = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{\frac{2(2)-1}{(2)^2}} = \frac{\sqrt{3}}{2} \rightarrow fog(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

ب) $f \quad f(g(\sqrt{2}))$ $\quad g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{(1+\sqrt{2})}{(1+\sqrt{2})} = \frac{\sqrt{2}+2}{1-2} = -(\sqrt{2}+2)$

$f(-\sqrt{2}-2) = [\underbrace{-\sqrt{2}-2}_{=-3/4}] = -4 \quad fog(\sqrt{2}) = -4$

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3 — (3)

$gof(\frac{\pi}{4}) = g(f(\frac{\pi}{4}))$ $\quad f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\underbrace{\sqrt{1-(\frac{\sqrt{2}}{2})^2}}_{\frac{\sqrt{2}}{2}} = \frac{1}{2}$

$gof(\frac{\pi}{4}) = \frac{1}{2}$

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4 — (3)

$g(x) = \{(4,6),(2,4),(6,8),(8,10)\} \quad f(x) = \{(4,5),(6,5),(8,12),(10,12)\}$

الف) $fog(x) = f(g(x)) = \{(4,5),(2,5),(6,12),(8,12)\}$ $\quad 4\rightarrow 6\rightarrow 5,\ 2\rightarrow 4\rightarrow 5,\ 6\rightarrow 8\rightarrow 12,\ 8\rightarrow 10\rightarrow 12$

ب) $gof(x) = g(f(x)) = \varnothing$ $\quad 4\rightarrow 5\rightarrow\times,\ 6\rightarrow 5\rightarrow\times,\ 8\rightarrow 12\rightarrow\times,\ 10\rightarrow 12\rightarrow\times$

ج) $fof(x) = f(f(x)) = \varnothing$ $\quad 4\rightarrow 5\rightarrow\times,\ 8\rightarrow 12\rightarrow\times,\ 6\rightarrow 5\rightarrow\times,\ 10\rightarrow 12\rightarrow\times$

د) $gog(x) = g(g(x)) = \{(4,8),(2,6),(6,10)\}$ $\quad 4\rightarrow 6\rightarrow 8,\ 2\rightarrow 4\rightarrow 6,\ 6\rightarrow 8\rightarrow 10,\ 8\rightarrow 10\rightarrow\times$

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5 — (2)

$f = \{(2,1)(3,2)(4,5)(1,7)\} \quad g = \{(1,2)(3,1)(a,3)(b,1)\}$

$(\underbrace{4}_{x=4},2)\in fog(x) \Rightarrow f(g(x)) = 2 \rightarrow g(x) = 3 \rightarrow x = a = 4 \qquad (a,b)\rightarrow(4,5)$

$(\underbrace{4}_{x=4},1)\in gof \rightarrow g(f(x)) = 1 \rightarrow f(x) = b$ یا $\cancel{3}$ $\xrightarrow{x=4} f(4) = 5 = b$

۳ توی برد f نیست

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6 — (3)

$f = ax+b \rightarrow f(f(x)) = a(ax+b)+b = a^2x+ab+b = 4x+3 \rightarrow a=2, b=1 \ /\ a=-2, b=-3$

$\rightarrow f(-1) = 2(-1)+1 = -1$ یا $-2(-1)-3 = -1$

$g(2x+3) = 3x-2 \quad 2x+3 = -1 \rightarrow x = -2 \rightarrow g(-1) = 3(-2)-2 = -8$

$gof(-1) = g(f(-1)) = -8$

فرناز محمد رضایی

$D_{gof} = \{x \in D_f : f(x) \in D_g\} \longrightarrow D_{gof} = ① \cap ② \cap ③ \longrightarrow D_{gof} = (0, +\infty) - \{8\}$

$D_f \Rightarrow x + |x| \geq 0 \rightarrow$ همواره برقرار $\rightarrow D_f = \mathbb{R}$ ③

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$D_g \rightarrow R - \{$ریشه‌های مخرج$\} \rightarrow \underset{x(x-4)}{x^2 - 4x \neq 0} \rightarrow x \neq 0, \ x \neq 4 \rightarrow D_g = \mathbb{R} - \{0, 4\}$ ۷

$D_{gof} = \{x \in \mathbb{R} , \sqrt{x+|x|} \in \mathbb{R} - \{0, 4\}\}$

$\sqrt{x+|x|} \neq 0 \rightarrow x + |x| \neq 0 \rightarrow |x| \neq -x \rightarrow x > 0$ ① $\mathbb{R} - \{8\}$ ②

$\sqrt{x+|x|} \neq 4 \rightarrow x + |x| \neq 16 \xrightarrow{x \geq 0} x + x \neq 16 \rightarrow x \neq 8$

$x < 0 \Rightarrow x - x \neq 16 \rightarrow 0 \neq 16$ همواره برقرار است

$D_{(f+g)of} = \{x \in D_f : f(x) \in D_{(f+g)}\}$

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$D_{(f+g)} = D_f \cap D_g \qquad D_f \Rightarrow 1 - x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$ ①

$D_g \rightarrow x \geq 0$ ② $\qquad D_{f+g} = ① \cap ② = [0, 1]$ ۸

$D_{(f+g)of} = \{x \in [-1, 1] , \sqrt{1-x^2} \in [0, 1]\} \xrightarrow{\cap} D_{(f+g)of} = [\cancel{0}, 1]$

$[-1, +1]$

الف) $\frac{3x+1}{x-2} = t \rightarrow tx - 2t = 3x + 1 \rightarrow 3x - tx = -2t - 1$

$x(3 - t) = -(2t + 1) \rightarrow x = \frac{-(2t+1)}{\cancel{3-t}} \ t-3 \rightarrow f(t) = 4\left(\frac{-(2t+1)}{3-t}\right) + 5 = \frac{-8t - 4 + 15 - 5t}{3-t}$

$= \frac{-13t + 11}{3-t} \rightarrow f(x) = \frac{-13x + 11}{\cancel{3-x}}$ $\frac{13x - 11}{x-3}$ ۹

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ب) $x + \frac{1}{x} = t \rightarrow (x + \frac{1}{x})^3 = x^3 + \frac{1}{x^3} + \underbrace{\cancel{3x^2 \frac{1}{x}} + \cancel{3x \frac{1}{x^2}}}_{3(x+\frac{1}{x})} = x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x})$

$\longrightarrow x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 - 3(x + \frac{1}{x}) \xrightarrow{x+\frac{1}{x}=t} f(t) = t^3 - 3t \rightarrow \boxed{f(x) = x^3 - 3x}$

$gof(x) = g(f(x))$

از آنجایی که تابع g محور x ها را در نقاطی به طول ۱ و $2\sqrt{2}$ قطع می کند
پس $g(1)=0$ و $g(2\sqrt{2})=0$ پس $f(x)$ باید مساوی ۱ و $2\sqrt{2}$ باشد
تا در تابع g محور x ها را قطع کند . سپس باید در تابع $f(x)$، x هایی را
که ۱ و $2\sqrt{2}$ می دهند را بیابیم تا طول نقاطی که $gof(x)$ محور x ها را قطع می کند بدست آید ۱۰

$g(1) = 0 , \ g(2\sqrt{2}) = 0 \longrightarrow f(x) = 2\sqrt{2}$ و $f(x) = 1 \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x = 2$

$\rightarrow x\sqrt{x} = 1 \rightarrow x = 1$

$2 - 1 = 1$

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