

۱۹,۱۵

$$f\left(\frac{\pi}{2}\right) = 2$$

$$f\left(\frac{\pi}{2}\right) = \cot \frac{\pi}{2} = \cot \frac{\pi}{2} = \sqrt{2}$$

$$\frac{\pi}{2} \leq 1$$

$$f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = 2$$

$$\frac{\pi}{2} = 1.57 > 1$$

۲

۱

$$f\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\pi}{2}\right) = 2 \cos^2 \frac{\pi}{2} = 2 \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\frac{1}{2} \leq \frac{1}{\pi} \rightarrow \pi \leq 2 \rightarrow f\left(\frac{1}{\pi}\right) = \sqrt{\frac{2(2)-1}{\pi^2}} = \sqrt{\frac{3}{\pi}} = \frac{\sqrt{3}}{\pi}$$

$$f\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} = -\sqrt{2}-2$$

$$f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -3$$

$$\sqrt{2} = 1.41 \rightarrow -2-\sqrt{2} = -3.41$$

$$[-3.41] = -4$$

۲

۳

$$g\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

۲

$$f \circ g(x) = \{(4,5), (2,5), (6,12), (8,12)\}$$

$$g \circ f(x) = \emptyset$$

الف)

$$4 \rightarrow 6 \rightarrow 5$$

$$4 \rightarrow 5 \rightarrow x$$

$$2 \rightarrow 2 \rightarrow 5$$

$$6 \rightarrow 5 \rightarrow x$$

$$6 \rightarrow 8 \rightarrow 12$$

$$8 \rightarrow 12 \rightarrow x$$

$$8 \rightarrow 10 \rightarrow 12$$

$$10 \rightarrow 12 \rightarrow x$$

$$ج) 4 \rightarrow 5 \rightarrow x$$

$$د) 4 \rightarrow 6 \rightarrow 8$$

$$6 \rightarrow 5 \rightarrow x$$

$$2 \rightarrow 4 \rightarrow 6$$

$$8 \rightarrow 12 \rightarrow x$$

$$6 \rightarrow 8 \rightarrow 10$$

$$10 \rightarrow 12 \rightarrow x$$

$$8 \rightarrow 10 \rightarrow x$$

$$ج) f \circ g(x) = \emptyset$$

$$د) g \circ g(x) = \{(4,8), (2,6), (6,10), (8,10)\}$$

۲

۴

$$(4,2) \in f \circ g$$

$$(4,1) \in g \circ f$$

$$f \circ g = \{(1,1), (2,2), (3,7), (4,7)\}$$

$$g \circ f = \{(2,2), (3,3), (4,1)\}$$

$$(a,b) = (4,5)$$

$$1 \rightarrow 2 \rightarrow 1$$

$$(4,2)$$

$$a = 4$$

$$2 \rightarrow 1 \rightarrow 2$$

$$3 \rightarrow 2 \rightarrow 3$$

$$4 \rightarrow 3 \rightarrow 4$$

$$b \rightarrow 1 \rightarrow 2$$

$$2 \rightarrow 1 \rightarrow 2$$

$$3 \rightarrow 2 \rightarrow 3$$

$$4 \rightarrow 3 \rightarrow 4$$

$$1 \rightarrow 2 \rightarrow 1$$

این مجموعه
مستقار بستنی دارد
از آن جایی که

$b=5$ پس $f \circ g$ راضی است

۲

۵

$f(x) = ax + b$ $f(f(x)) = f(ax+b) = a(ax+b) + b = a^2x + ab + b \rightarrow f \circ f = a^2x + ab + b$ $f(x) = x + r + 1 \rightarrow x = -1 \rightarrow -r + 1 = -1$ $f(x) = -rx - r \rightarrow x = -1 \rightarrow r - r = -1$ $rx + r = -1 \rightarrow x = -1$ $f(-1) = 1 \rightarrow g \circ f(-1) = -1$ $g(-1) = r(-r) - r = -1$	6
$M = g \circ f : D_f \rightarrow D_g \cap \mathbb{R} \Rightarrow D_M = (0, +\infty) - \{1\}$ $\{x \mid f(x) \in D_g, x \in D_f\}$ $D_g \Rightarrow x^2 - x \neq 0 \rightarrow x \neq 0, x \neq 1 \rightarrow \sqrt{x+ x } \neq 0 \rightarrow x+ x \neq 0 \rightarrow x \neq -x \rightarrow x > 0$ $x(x-1) \neq 0 \rightarrow \sqrt{x+ x } \neq 0 \rightarrow x+ x \neq 0 \rightarrow x \geq 0 \rightarrow x \neq 1 \rightarrow x \in \mathbb{R} - \{1\}$ $D_f \Rightarrow x+ x \geq 0 \rightarrow \mathbb{R} \rightarrow x \in \mathbb{R}$	7 $\mathbb{R} - \{1\}$
$f+g = \sqrt{x} + \sqrt{1-x} \rightarrow M = (f+g) \circ f = \mathbb{R} \cap \mathbb{R} \Rightarrow D_M = [0, 1]$ $\{x \mid x \in D_f, f(x) \in D_{f+g}\}$ $D_f \Rightarrow 1-x^2 \geq 0$ $x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \rightarrow [-1, 1]$ $D_{f+g} \Rightarrow \begin{cases} x \geq 0 \\ 1-x^2 \geq 0 \end{cases} \Rightarrow 0 \leq x \leq 1 \rightarrow [0, 1]$ $D(f+g) \cap D_f = [0, 1]$	8
$f\left(\frac{r+1}{x-r}\right) = rx + a$ $\frac{r+1}{x-r} = t \rightarrow x = \frac{-rt-1}{t-r} = \frac{rt+1}{t-r} \rightarrow f(t) = r\left(\frac{rt+1}{t-r}\right) + a$ $\frac{rt+1}{t-r} + a = \frac{r^2t-1}{t-r} \Rightarrow \frac{rt+1}{t-r} + a = \frac{r^2t-1}{t-r}$ $f(x) = \frac{rx-1}{x-r}$ $x + \frac{1}{x} = t \rightarrow t^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \rightarrow t^2 - 2 = x^2 + \frac{1}{x^2} \rightarrow t^2 - 2 = \frac{x^4 + 1}{x^2} \rightarrow t^2 - 2 = \frac{x^4 + 1}{x^2}$ $f(x) = \frac{rx-1}{x-r}$	9
$g(f(x)) = 0 \rightarrow f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x^2 = 1 \rightarrow x = 1$ $f(x) = r\sqrt{x} \rightarrow x\sqrt{x} = r\sqrt{x} \rightarrow x = r$	10