(۱) $\cot(\frac{\pi}{4}) = \sqrt{c} \rightarrow f(\sqrt{c}) = \sqrt{c+1} = 1$ (۱۶)

$f \circ f(\frac{\pi}{2}) = 1$ (۲)

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(۲) الف

$g(\frac{\pi}{2}) = 2\cos^2(\frac{\pi}{4}) = \frac{1}{2}$

$F(\frac{1}{2}) = \sqrt{\frac{1}{2} - \frac{1}{4}} \Rightarrow f(x) = \sqrt{2x - x^2}$

$f(\frac{1}{4}) = \sqrt{2 \times \frac{1}{4} - \frac{1}{16}} = \frac{\sqrt{7}}{4}$ (۲)

$f(\frac{1}{4}) = \frac{\sqrt{7}}{4} \rightarrow f \circ g(\frac{\pi}{2}) = \frac{\sqrt{7}}{4}$

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(۳) ب

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{\sqrt{2}+2}{-1} = -2-\sqrt{2}$

$f(-2-\sqrt{2}) = [-2-\sqrt{2}] + 2 = [-\sqrt{2}] = -3$

$f \circ g(\sqrt{2}) = -3$

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(۴) $F(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \sqrt{\frac{1}{2} - 1} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$ (۲)

$g \circ f(\frac{\pi}{4}) = \frac{1}{2}$

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(۵)

$f \circ g(x) = g(x) \Rightarrow f \circ g(x) = \{(3,0), (5,2), (7,12), (9,4)\}$

$3 \rightarrow 6 \rightarrow 0$
$5 \rightarrow 3 \rightarrow 2$
$7 \rightarrow 7 \rightarrow 6$
$9 \rightarrow 10 \rightarrow 7$

(۲)

$g \circ f(x) \rightarrow$ $3 \rightarrow 0 \rightarrow$ ✗
$6 \rightarrow 1 \rightarrow$ ✗
$7 \rightarrow 12 \rightarrow$ ✗
$10 \rightarrow 11 \rightarrow$ ✗

$\Rightarrow g \circ f(x) = [\emptyset]$

(۴۰/۲) $f \circ f(x) \longrightarrow$ $2 \to 5 \not\to$, $4 \to 5 \not\to$, $8 \to 12 \not\to$, $10 \to 12 \not\to$ — غیر

(۴/۳) $g \circ g(x) \longrightarrow$ $2 \to 4 \to 8$, $4 \to 2 \to 4$, $6 \to 8 \to 10$, $8 \to 10 \not\to$

$\to g \circ g(x) = \{(2,8), (4,4), (6,10)\}$

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۵ $f \circ g \longrightarrow$ $1 \to 4 \to 1$, $c \to 1 \to 7$, $a \to c \to 4$, $b \to 1 \to 7$

$\to f \circ g = \{(1,1)(c,7)(a,4)(b,7)\}$

$\swarrow a = 4$ (۲)

$g \circ f \longrightarrow$ $4 \to 1 \to 4$, $c \to 4 \not\to$, $8 \to 0 \to b$, $1 \to 7 \not\to$

$\to g \circ f = \{(4,4), (8,b)\}$

$\swarrow b = 5$

$\boxed{(a,b) = (4,5)}$

---

(۶) $f(x) = ax + b \quad g(x) = cx + d$

$f(f(x)) = a(ax+b) + b = a^2x + ab + b = 4x + c$

$a = 2 \to b = +1$

$a = -2 \to b = -3$

$g(2x + c) = cx - 2 = 2xc + c^2 + d = c^2 x - 2$

$2xc = c^2 x \to c = 1,5$ (۲)

$c^2 + d = -2 \to d = -7,5$

$g(x) = 1,5x - 7,5$

$f(x) = 2x + 1 \to f(-1) = -1$

$\boxed{g(f(-1)) = -8}$

$f(x) = -2x - 3 \to f(-1) = -1$

$\boxed{g(f(-1)) = -8}$

(۱) $g \circ f(x)$

$D_f = R \quad R_f = [0, +\infty)$

$D_g \to R - [\sqrt{2}, 0] \to \sqrt{2} = \sqrt{2x} \to x = 1$

$0 = \sqrt{2x} \to x = 0$

$|x+1| \neq 0 \to x \neq -1$

$\sqrt{|x+1|} \neq 2 \to x \neq 1$

$D_{gf} = (0, +\infty) - \{1\}$

$R = \{1, 0\}$

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(۲) $D_{f+g} = D_f \cap D_g$

$D_f = x \geq 0$

$D_g = -1 \leq x \leq 1$

$\to D_{f+g} = [0, 1]$

$D_{(f+g) \circ f} \to 0 \leq \sqrt{1-x^2} \leq 1$

$-1 \leq x \leq 1$

$\to D_{(f+g) \circ f} = [-1, 1]$

---

(۳) $f\left(\frac{2x+1}{x-2}\right) = 4x + 5$

$\frac{2x+1}{x-2} = t \to 2x + 1 = tx - 2t \to x(2-t) = -1 - 2t$

$x = \frac{-1-2t}{2-t} = \frac{2t+1}{t-2}$

$\to f(t) = 4\left(\frac{2t+1}{t-2}\right) + 5 = \frac{8t + 4 - 10 + 5t}{t-2}$

$f(t) = \frac{13t - 11}{t-2} \to f(x) = \frac{13x - 11}{x-2}$

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(۴) $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

$x + \frac{1}{x} = t \to x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2(x + \frac{1}{x})$

$f(t) = t^2 - 2t \to f(x) = x^2 - 2x$

$g(1) = 0$

$g(2\sqrt{2}) = 0$

$g(x\sqrt{x}) = 0$

$x\sqrt{x} = 1 \to x = 1$

$x\sqrt{x} = 2\sqrt{2} \to x = 2$

(۵) $x_1 - x_2 = 2 - 1 = 1 \to \Delta x = 1$