

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$$

$$f \circ f\left(\frac{1}{\sqrt{2}}\right) = f\left(f\left(\frac{1}{\sqrt{2}}\right)\right) = ?$$

$$\hookrightarrow \cot \frac{\pi x}{2} \xrightarrow{x=\frac{1}{\sqrt{2}}} \cot \frac{\pi}{2\sqrt{2}} \rightarrow \sqrt{2}$$

$$\longrightarrow \sqrt{x^2 + 1} \xrightarrow{x=\sqrt{2}} \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$$

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$$\text{الف) } f\left(\frac{1}{n}\right) = \sqrt{\frac{r_n - 1}{n^2}} \quad , \quad g(n) = r \cos^n n$$

$$n = \frac{1}{n} \Rightarrow f(n) = \sqrt{\frac{\frac{r}{n} - 1}{\frac{1}{n^2}}} = \sqrt{\frac{r - n}{\frac{1}{n}}} = \sqrt{r - n} \cdot \sqrt{n} = \sqrt{r - n^2}$$

$$f \circ g\left(\frac{\pi}{2}\right) = f\left(g\left(\frac{\pi}{2}\right)\right)$$

$$g\left(\frac{\pi}{2}\right) = r \cos^n n = r \left(\frac{1}{r}\right)^r = \frac{1}{r}$$

$$\hookrightarrow f\left(\frac{1}{r}\right) = \sqrt{r \left(\frac{1}{r}\right) - \left(\frac{1}{r}\right)^2} = \sqrt{1 - \frac{1}{r}} = \sqrt{\frac{r-1}{r}}$$

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$$\text{ب) } f(n) = [n] \quad g(n) = \frac{n}{1-n}$$

$$f \circ g(\sqrt{2}) = f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \Rightarrow f\left(\frac{\sqrt{2}}{1-\sqrt{2}}\right) = \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] = [-1, 0] = -1$$

$$\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2} + 2}{1-2} = -\sqrt{2} - 2$$

$$f(n) = \sin n$$

$$g(n) = n \sqrt{1-n^2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = g\left(f\left(\frac{\pi}{2}\right)\right) \Rightarrow f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1 \Rightarrow g(1) = 1 \sqrt{1-1^2} = 0$$

$$\Rightarrow \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} \Rightarrow \frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

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$$g(n) = \{(1, 4), (2, 5), (4, 8), (8, 16)\} \quad f(n) = \{(1, 2), (2, 4), (4, 8), (8, 16)\}$$

$$\text{الف) } f \circ g(n) \Rightarrow \begin{matrix} 1 \rightarrow 4 \rightarrow 8 \\ 2 \rightarrow 5 \rightarrow 16 \\ 4 \rightarrow 8 \rightarrow 16 \\ 8 \rightarrow 16 \rightarrow 16 \end{matrix} \rightarrow f \circ g(n) = \{(1, 8), (2, 16), (4, 16), (8, 16)\}$$

$$\text{ب) } g \circ f(n) \Rightarrow \begin{matrix} 1 \rightarrow 2 \rightarrow 4 \\ 2 \rightarrow 4 \rightarrow 8 \\ 4 \rightarrow 8 \rightarrow 16 \\ 8 \rightarrow 16 \rightarrow 16 \end{matrix} \rightarrow g \circ f(n) = \emptyset$$

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$$f = \{(1, 1), (2, 2), (3, 3), (4, 4)\} \quad g(n) = \{(1, 2), (2, 1), (3, 3), (4, 4)\}$$

$$f \circ g(n) \Rightarrow \begin{matrix} 1 \rightarrow 2 \rightarrow 2 \\ 2 \rightarrow 1 \rightarrow 1 \\ 3 \rightarrow 3 \rightarrow 3 \\ 4 \rightarrow 4 \rightarrow 4 \end{matrix} \rightarrow f \circ g(n) = \{(1, 2), (2, 1), (3, 3), (4, 4)\}$$

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$$f(f(x)) = \{x+1\} \Rightarrow f(x) = a(x+b) \Rightarrow a(a(x+b)+b) = \{x+1\} \Rightarrow a^2x + ab + b = \{x+1\}$$

$$g(x+1) = x-1 \Rightarrow a = -1 \Rightarrow b = 1$$

$$g(f(x)) = g(\{x+1\}) = x-1$$

$$g(x+1) = x-1 \Rightarrow g(-1) = x-1 \Rightarrow g(-1) = -1$$

$$g(-x-1) = x-1 \Rightarrow g(-1) = x-1 \Rightarrow g(-1) = -1$$

$$f(x) = \sqrt{x+1}$$

$$g(x) = \frac{1}{x^2-1}$$

$$g(f(x)) = \frac{1}{x^2-1} \Rightarrow \frac{1}{x^2-1} = \frac{1}{x^2-1} \Rightarrow \frac{1}{x^2-1} = \frac{1}{x^2-1}$$

$$x > 0 \Rightarrow \frac{1}{x^2-1} = \frac{1}{x^2-1} \Rightarrow \frac{1}{x^2-1} = \frac{1}{x^2-1}$$

$$x < 0 \Rightarrow \frac{1}{x^2-1} = \frac{1}{x^2-1} \Rightarrow \frac{1}{x^2-1} = \frac{1}{x^2-1}$$

$$f(x) = \sqrt{1-x} \Rightarrow 1-x \geq 0 \Rightarrow 1 \geq x \Rightarrow 1 \geq x \geq -1$$

$$g(x) = \sqrt{x} \Rightarrow x \geq 0$$

$$D_f \cap D_g = [0, 1]$$

$$g(f(x)) = g(\sqrt{1-x}) \Rightarrow D_f = [0, 1]$$

$$1-x \geq 0 \Rightarrow 1 \geq x \geq -1$$

$$f\left(\frac{x+1}{x-1}\right) = \{x+1\} \Rightarrow \frac{x+1}{x-1} = t \Rightarrow x+1 = t(x-1) \Rightarrow x+1 = tx-t \Rightarrow x+1 = tx-t$$

$$x(t-1) = -t-1 \Rightarrow x = \frac{-t-1}{t-1}$$

$$f(x) = \frac{-t-1}{t-1}$$

$$f\left(\frac{x+1}{x-1}\right) = \frac{x+1}{x-1} = \frac{t+1}{t-1} = \frac{t+1}{t-1}$$

$$f(x) = \frac{x+1}{x-1}$$



$$g(1) = 1$$

$$g(\sqrt{x}) = 1$$

$$g(x\sqrt{x}) = 0$$

$$x\sqrt{x} = 1 \Rightarrow x = 1$$

$$x\sqrt{x} = x\sqrt{x} \Rightarrow x = x$$