نام و نام خانوادگی: کیمیا رجبی | شماره یا شناسه: ۹ | ۱۸٫۵ | نوبت یازدهم دختر A

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$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x<1\\ \sqrt{x^2+1} & x>1\end{cases} \qquad fof\left(\frac{1}{2}\right)=f\left(f\left(\frac{1}{2}\right)\right)=?$$

$$\hookrightarrow \cot\frac{\pi}{4}x \xrightarrow{x=\frac{1}{2}} \cot\frac{\pi}{8}\to\sqrt{2}$$

$$\longrightarrow \sqrt{x^2+1} \xrightarrow{x=\sqrt{2}} \sqrt{(\sqrt{2})^2+1}=\sqrt{3}$$

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الف) $f\left(\frac{1}{x}\right)=\sqrt{\frac{x-2x^2}{x^2}}$ ، $g(x)=2\cos^2 x$

$$x=\frac{1}{x} \Rightarrow f(x)=\sqrt{\frac{\frac{1}{x}-1}{\frac{1}{x^2}}}=\sqrt{\frac{\frac{1-x}{x}}{\frac{1}{x^2}}}=\sqrt{1x-x^2}$$

$$fog\left(\frac{\pi}{3}\right)=f\left(g\left(\frac{\pi}{3}\right)\right)$$

$$g\left(\frac{\pi}{3}\right)=2\cos^2\frac{\pi}{3}=2\left(\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\hookrightarrow f\left(\frac{1}{2}\right)=\sqrt{2\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

ب) $f(x)=[x]$ $g(x)=\frac{x}{1-x}$

$$fog(\sqrt{2})=f(g\sqrt{2}) \Rightarrow g(\sqrt{2})=-\sqrt{2}-2 \Rightarrow f(-\sqrt{2}-2)=\left[-\sqrt{2}-2\right]=[-3.4]=-4$$

$$\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{1-2}=-\sqrt{2}-2$$

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$$f(x)=\sin x$$

$$g(x)=x\sqrt{1-x^2}$$

$$gof\left(\frac{\pi}{3}\right)=g(f(\tfrac{\pi}{3})) \Rightarrow f\left(\frac{\pi}{3}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2} \Rightarrow g\left(\frac{\sqrt{3}}{2}\right)=x\sqrt{1-x^2}$$

$$\Rightarrow \frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} \Rightarrow \frac{\sqrt{3}}{2}\sqrt{\frac{1}{4}}=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}=\frac{1}{4}$$

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$$g(x)=\{(3,4),(2,3),(4,7),(7,9),(10,10)\} \qquad f(x)=\{(5,3),(6,5),(7,14),(9,12),(12,10)\}$$

الف) $fog(x)$: $3\to5\to9$، $2\to3\to12$، $4\to7\to14$، $7\to9\to12$، $10\to10\to10$

$$\Rightarrow fog(x)=\{(3,9),(2,12),(4,14),(7,12),(10,10)\}$$

ب) $gof(x)$: $5\to6\to7$، $3\to5\to9$، $6\to7\to14$، $9\to12\to10$، $12\to10\to12$

$$\Rightarrow gof(x)=\emptyset$$

ج) $fof(x)$: $3\to5\to6$، $5\to6\to7$، $6\to7\to14$، $9\to12\to10$، $12\to10\to12$

$$\Rightarrow fof(x)=\emptyset$$

د) $gog(x)$: $4\to7\to9$، $3\to4\to7$، $7\to9\to10$، $9\to10\to12$، $10\to12\to10$

$$\Rightarrow gog(x)=\{(3,9),(2,7),(4,10),(7,12),(10,10)\}$$

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$$f=\{(1,3),(3,2),(5,4),(4,1)\} \qquad g(x)=\{(1,4),(2,3),(3,5),(4,2),(5,1)\}$$

$fog(x)$: $1\to2\to1$، $4\to1\to3$، $3\to4\to2$، $5\to1\to4$

$$(3,2)\in fog(x) \Rightarrow (a,2)=(3,b) \Rightarrow a=3,\ b=4$$

$gof(x)$: $2\to1\to4$، $4\to2\to3$، $3\to5\to1$، $5\to4\to2$، $1\to3\to5$

$$(4,1)\in gof(x) \Rightarrow (3,a)=(4,b) \Rightarrow a=1,\ b=3$$

$$(a,b)=(3,5)$$

(۵)

$f(f(x)) = 4x + 1$ $\Rightarrow$ $f(x) = ax + b$ $\Rightarrow$ $a(ax+b)+b = a^2x + ab + b = 4x + 1$

$\downarrow$ $a^2 = 4 \rightarrow a = \pm 2$

$g(2x+1), x-2$ $\quad 2x+1 = -1 \rightarrow x = -1$ $\quad 2x-2 \xrightarrow{x=-1} -4$

$2x+1 \xleftarrow{a=2} 2b+b = 1 \Rightarrow b = 1$

$-2x-1 \xrightarrow{a=-2} -2b+b = -b = 1 \Rightarrow b = -1$

$gof(x) = g(f(-1)) = 2^{2x} - 2$

$\downarrow g(2x+1) = 2^x - 2 \Rightarrow g(-1) = 2^{-1} - 2 \Rightarrow g(-1) = -\frac{3}{2}$ $\quad -4$

$g(-2x-1) = 2^{-x} - 2 \Rightarrow g(-1) = 2^{x} - 2 \Rightarrow g(-1) = 0$

$\rightarrow gof(-1) = -4$

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$f(x) = \sqrt{x + |x|}$ $\quad gof(x) = ? \Rightarrow g(f(x))$

$g(x) = \frac{1}{x^2 - 4x}$

$g(\sqrt{x+|x|}) = \frac{1}{x^2 - 4x} \Rightarrow \frac{1}{(\sqrt{x+|x|})^2 - 4(\sqrt{x+|x|})}$

$\frac{1}{x+|x| - 4\sqrt{x+|x|}} \Rightarrow x > 0 \Rightarrow \frac{1}{2x - 4\sqrt{2x}} \Rightarrow \frac{1}{2(x - 2\sqrt{2x})} \Rightarrow D_f = (0, +\infty) - \{4\}$

$\Downarrow$ $x < 0$ $\quad \frac{1}{-0 - 4\sqrt{0}} = \infty \Rightarrow X$

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7

$f(x) = \sqrt{1 - x^2} \Rightarrow 1 - x^2 \geq 0 \Rightarrow 1 \geq x^2 \Rightarrow 1 \geq x \geq -1$ $\rightarrow D_f \cap D_g = [0, 1]$

$g(x) = \sqrt{x} \Rightarrow x \geq 0$

$gof(f(x)) = g(f(\sqrt{1-x^2})) \Rightarrow D_f = [0, 1]$ $\quad 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow x^2 \geq 0$

$1 - x^2 \geq 0 \rightarrow 1 \geq x \geq -1$

$D_{(f+g)of} = [-1, 1]$

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1,0

$f\left(\frac{x^2+1}{x-2}\right) = 4x + 2 \Rightarrow \frac{x^2+1}{x-2} = t \Rightarrow x^2 + 1 = tx - 2t \Rightarrow x^2 - tx = -2t - 1$

$x(x - t) = -2t - 1$

$x = \frac{-2t-1}{x-t}$

$f(x) = \frac{-2x-1}{x-x}$

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right) = t\left(x^2 + \frac{1}{x^2} + 2 - 1\right) = t((t)^2 - 1) = t^3 - t$

$x + \frac{1}{x} = t$

$f(x) = x^3 - x$

الف) $F\left(\frac{2t+1}{t-2}\right) + 2$

9

1,0

$g(1) = 1$

$g(2\sqrt{2}) = 2$

$g(x\sqrt{x}) = 0$

$\downarrow$

$x\sqrt{x} = 1 \Rightarrow x = 1$

$x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2$ $\rightarrow 2 - 1 = 1$

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