نام و نام خانوادگی بهار شهیدیان تکلیف ۹ (۲۰) گروه یازدهم دختر A

۱ (۲)

$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$ $f(\frac{2}{3})$ $\cot \frac{\pi \cdot \frac{2}{3}}{4} = \frac{\pi}{6} \to \cot\frac{\pi}{6} \to \sqrt{3}$

$\to f(\sqrt{3}) \to \sqrt{(\sqrt{3})^2+1} = 2$

$fof(\frac{2}{3}) \to f(f(\frac{2}{3})) = 2$

۲ (۲)

الف) $f(\frac{1}{2}) = \sqrt{\frac{2x-1}{x^2}} \to \sqrt{\frac{4-1}{4}}$

$g(x) = 2\cos^2 x \to 2\cos^2\frac{\pi}{3} = \frac{1}{2}$

$fog(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

ب) $f(x) = [x] \to [-3.4] = -4$

$g(x) = \frac{x}{1-x} \to \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -3.4$

$fog(\sqrt{2}) = -4$

۳ (۲)

$f(x) = \sin x \to \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$g(x) = x\sqrt{1-x^2} \to \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

$gof(\frac{\pi}{4}) = \frac{1}{2}$

۴ (۲)

الف) $fog(x)$ ، $f(g(x))$

$4 \to 6 \to 5$ / $3 \to 4 \to 5$

$6 \to 8 \to 12$ / $8 \to 10 \to 12$

$fog(x) = \{(4,5)(3,5)(6,12)(8,12)\}$

ب) $gof(x)$

$4 \to 5 \to X$ / $6 \to 5 \to X$

$8 \to 12 \to X$ / $10 \to 12 \to X$

$gof = \varnothing$

ج) $fof(x)$

$4 \to 5 \to X$ / $8 \to 12 \to X$

$6 \to 5 \to X$ / $10 \to 12 \to X$

$fof = \varnothing$

د) $gog(x)$

$4 \to 6 \to 8$

$3 \to 4 \to 6$

$6 \to 8 \to 10$

$8 \to 10 \to X$

$gog(x) = \{(4,8)(3,6)(6,10)\}$

۵ (۲)

$f = \{(2,1)(3,2)(4,5)(1,7)\}$ $g(x) = \{(1,2)(3,1)(a,4)(b,1)\}$

$(4,2) \in fog$

$(4,1) \in gof$

$fog \to \begin{cases} 1 \to 2 \to 1 \\ 3 \to 1 \to 7 \\ a \to 4 \to 2 \\ b \to 1 \to 7 \end{cases} \Rightarrow a = 4$

$(4,5)$

$gof \to \begin{cases} 2 \to 1 \to 2 \\ 3 \to 2 \to X \\ 4 \to 5 \to 1 \\ 1 \to 7 \to X \end{cases} \Rightarrow b = 5$

$f(f(x)) = 4x+3 \rightarrow a(ax+b)+b = 4x+3 \rightarrow a^2x+ab+b = 4x+3$ → $a=\pm 2$, $b=-1,3$ 6

$g(2x+4) = 3x-2 \rightarrow a(2x+4)+b = 2ax+4a+b = 3x-2 \rightarrow a=\frac{3}{2} , b=-\frac{13}{2}$

$g\circ f(-1) = -8 \Rightarrow f = 2x+1 , g = \frac{3}{2}x - \frac{13}{2}$

$f(-1) = -2+1 = -1 \quad g(-1) = -\frac{3}{2} - \frac{13}{2} = -\frac{16}{2} = -8$

(2)

$f(x) = \sqrt{x+|x|} \qquad g(x) = \frac{1}{x^2-4x}$ 7

$D_{g\circ f} = \{x \in D_f , f(x) \in D_g\} \rightarrow \begin{cases} x \in R \\ \sqrt{x+|x|} \in R-\{0,4\} \end{cases} \rightarrow D_{g\circ f} = (0,\infty) - \{8\}$

(2)

$D_f = x+|x| \geq 0 \rightarrow R$

$D_g = x^2-4x \neq 0 \rightarrow x(x-4)\neq 0 \rightarrow \begin{matrix} x\neq 0 \\ x\neq 4 \end{matrix} \Big\} \begin{matrix} x+|x| \neq 0 \rightarrow x \neq (-\infty,0] \\ x+|x| \neq 16 \rightarrow x \neq 8 \end{matrix}$

$f(x) = \sqrt{1-x^2} , g(x) = \sqrt{x}$ 8

$D_{(f+g)\circ f} \{x \in D_f , f(x) \in D_{f+g}\} \rightarrow \{x \in [-1,1] , x \in [-1,1]\} \quad D = [-1,1]$

$D_{f+g} = D_f \cap D_g \Rightarrow [-1,1] \cap [0,\infty) = [0,1]$ (2)

$D_g = x \geq 0 , D_f = [-1,1]$ $\qquad \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 1 \geq x^2 \geq 0 \rightarrow x \in [-1,1]$

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$ 9

$\frac{3x+1}{x-2} = t \rightarrow x = \frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$

$\rightarrow f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 \rightarrow \frac{8t+4}{t-3}+5$

$\rightarrow f(x) = \frac{8x+4}{x-3}+5$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3}$

$x+\frac{1}{x} = t \rightarrow \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$f(t) = t^3-3t$

$f(x) = x^3-3x$

(2)

$f(x) = x\sqrt{x}$ 10

$g\circ f(x) \rightarrow g(f(x)) = 0$, $g(1) = 0 , g(2\sqrt{2}) = 0$ $\Big\} \rightarrow$ $f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x_1 = 1$

$f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x_2 = 2$

(2)

اختلاف طول نقاط $= x_2 - x_1 = 2-1 = 1$

نمودار gof محور عرض‌ها را قطع
می‌کند