

$$L(L(\frac{\pi}{4})) \rightarrow L(\frac{\pi}{4}) = \cos(\frac{\pi}{4} \times \frac{\pi}{2}) \rightarrow \cos(\frac{\pi}{2}) = \sqrt{2}$$

$$L(\sqrt{2}) \Rightarrow \sqrt{(\sqrt{2})^2 + 1} = 2$$

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$$\text{الف) } L(g(\frac{\pi}{4})) \rightarrow g(\frac{\pi}{4}) \rightarrow \sqrt{\cos^2 \frac{\pi}{4}} = \frac{1}{\sqrt{2}} \quad L(\frac{1}{\sqrt{2}}) = \sqrt{\frac{2-1}{2}} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow L(g(\sqrt{2})) \rightarrow g\sqrt{2} = \frac{\sqrt{2}}{1+\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$$

$$L(-\sqrt{2}-2) = [-2-\sqrt{2}] \rightarrow -2[-\sqrt{2}] = -2 \times -2 = 2$$

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$$g(L(\frac{\pi}{2})) \Rightarrow L(\frac{\pi}{2}) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$g(\frac{\sqrt{2}}{2}) \Rightarrow \frac{\sqrt{2}}{2} \sqrt{1-\frac{2}{2}} \rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{2}$$

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$$\text{الف) } \begin{array}{l} \varepsilon \xrightarrow{g(x)} \gamma \xrightarrow{L(x)} \omega \\ \gamma \xrightarrow{g(x)} \varepsilon \xrightarrow{L(x)} \omega \\ \gamma \xrightarrow{g(x)} \gamma \xrightarrow{L(x)} \gamma \\ \gamma \xrightarrow{g(x)} \omega \xrightarrow{L(x)} \gamma \end{array} \quad L \circ g(x) = \{(\varepsilon, \omega), (\gamma, \omega), (\gamma, \gamma), (\omega, \gamma)\}$$

$$\Rightarrow \begin{array}{l} \varepsilon \xrightarrow{L(x)} \omega \\ \gamma \xrightarrow{L(x)} \omega \\ \gamma \xrightarrow{L(x)} \gamma \\ \omega \xrightarrow{L(x)} \gamma \end{array} \quad g \circ L(x) = \emptyset$$

$$\text{ج) } \begin{array}{l} \varepsilon \xrightarrow{L(x)} \omega \\ \gamma \xrightarrow{L(x)} \omega \\ \gamma \xrightarrow{L(x)} \gamma \\ \omega \xrightarrow{L(x)} \gamma \end{array} \quad L \circ L(x) = \emptyset$$

$$\Rightarrow \begin{array}{l} \varepsilon \xrightarrow{g(x)} \gamma \rightarrow \gamma \\ \gamma \xrightarrow{g(x)} \varepsilon \rightarrow \gamma \\ \gamma \xrightarrow{g(x)} \gamma \rightarrow \omega \\ \omega \xrightarrow{g(x)} \omega \rightarrow \gamma \end{array} \quad g \circ g(x) = \{(\varepsilon, \gamma), (\gamma, \gamma), (\gamma, \omega), (\omega, \gamma)\}$$

$$L(g(x)) \rightarrow \varepsilon \xrightarrow{g(x)} \left\{ \begin{array}{l} \gamma \\ \omega \end{array} \right\} \xrightarrow{L(x)} \gamma \quad \underline{a \neq \varepsilon}$$

$$(\varepsilon, \omega)$$

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$$g(L(x)) \rightarrow \varepsilon \rightarrow \left\{ \begin{array}{l} \gamma \\ \omega \end{array} \right\} \rightarrow \gamma \quad \underline{b \neq \omega}$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b = \varepsilon x + \tau$$

$$a^2 = \varepsilon \rightarrow a = \pm \sqrt{\varepsilon} \quad \begin{cases} +\sqrt{\varepsilon} \rightarrow \varepsilon x + \tau b + b = \varepsilon x + \tau b \\ -\sqrt{\varepsilon} \rightarrow \varepsilon x + \tau b + b = \varepsilon x - b \end{cases} \quad \begin{aligned} \varepsilon x + \tau b = \varepsilon x + \tau &\rightarrow b = 1 \\ \varepsilon x - b = \varepsilon x + \tau &\rightarrow b = -\tau \end{aligned}$$

$$f(x) = \varepsilon x + 1 \quad f(-1) = \begin{cases} -\varepsilon + 1 = \tau \\ -\varepsilon - \tau = -\tau \end{cases} \quad \begin{aligned} g(-\tau) &= -\tau - \tau = -1 \\ g(-\tau) &= -(\tau - \tau) = -1 \end{aligned}$$

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$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \quad x + |x| \geq 0 \rightarrow x \geq 0$$

$$D_{g \circ f} = \left\{ \begin{matrix} x \geq 0 \\ x \geq 0 : \sqrt{x+|x|} \neq \{0, 2\} \end{matrix} \right\} \quad D_{g \circ f} = [0, +\infty) - \{0, 2\}$$

$$x \neq 0 \quad \begin{aligned} x + |x| &\neq 0 \\ \tau x &\neq \varepsilon \\ x &\neq \tau \end{aligned}$$

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$$D_{f \circ g \circ f} = \{x \in D_f : f(x) \in D_{f \circ g}\} \quad 1 - x^2 \geq 0 \quad \frac{1}{1-x^2} \quad D_f = [-1, 1]$$

$$\textcircled{1} \quad = \{[-1, 1] : f(x) \in \{0, 1\}\}$$

$$\downarrow$$

$$\sqrt{1-x^2} = 0 \rightarrow x^2 = 1 \quad x = \pm 1 \quad \textcircled{2} \quad \{+1, -1, 0\}$$

$$\sqrt{1-x^2} = 1 \rightarrow x^2 = 0 \quad x = 0$$

$$\Rightarrow D_{f \circ g \circ f} = [-1, 1]$$

$$f(x) = \{(1, 0), (0, 1)\}$$

$$g(x) = \{(1, 1), (0, 0)\} \quad D_{f \circ g} = \{1, 0\}$$

$$f \circ g = \{(1, 1), (0, 1)\}$$

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$$\text{a)} \quad \frac{x+1}{x-2} = t \rightarrow x+1 = t(x-2) \rightarrow x(t-2) = 2t-1 \quad x = \frac{2t+1}{t-2}$$

$$f(t) = \varepsilon \left(\frac{2t+1}{t-2} \right) + \omega \rightarrow f(x) = \varepsilon \left(\frac{x+1}{x-2} \right) + \omega \rightarrow f(x) = \frac{x+1}{x-2} + \omega$$

$$\text{b)} \quad x + \frac{1}{x} = t \quad \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + \frac{2}{x} + \frac{2}{x} \rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - \frac{4}{x^2}$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - \frac{4}{x^2} \rightarrow f(t) = t^2 - \frac{4}{x^2} \rightarrow f(x) = x^2 - \frac{4}{x^2}$$

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$$f(x) = 1 \rightarrow x\sqrt{x} = 1 \xrightarrow{\sqrt{x}=t} t^3 = 1 \rightarrow t^2 = 1 \rightarrow t = 1 \rightarrow \sqrt{x} = 1 \rightarrow x = 1$$

$$f(x) = \tau\sqrt{x} \rightarrow x\sqrt{x} = \tau\sqrt{x} \rightarrow x = \tau$$

$$\tau - 1 = 1$$

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