نام و نام خانوادگی ..[illegible].. پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ....

۱۷,۲۵

۱	$f(f(\frac{3}{4})) \to f(\frac{3}{4}) = \cot(\frac{3}{4}\times\frac{\pi}{4}) \to \cot(\frac{3\pi}{12}) = \sqrt{3}$ $f(\sqrt{3}) \Rightarrow \sqrt{(\sqrt{3})^2+1} = 2$ (۲)
۲	الف) $f(g(\frac{\pi}{3})) \to g(\frac{\pi}{3}) \to 2\cos^2\frac{\pi}{3} = \frac{1}{2}$ $\quad f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$ ب) $f(g(\sqrt{2})) \to g(\sqrt{2}) = \frac{2}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$ $f(-\sqrt{2}-2) = [-2-\sqrt{2}] \to \cancel{-2}\ [-\sqrt{2}] = -2\times-2 = 4$ -۴ (۱,۷۵)
۳	$g(f(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $g(\frac{\sqrt{2}}{2}) \Rightarrow \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} \to \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} = \frac{2}{4}$ (۳)
۴	الف) $4 \xrightarrow{g(x)} 6 \xrightarrow{f(x)} 5$; $2 \xrightarrow{g(x)} 4 \xrightarrow{f(x)} 5$; $6 \xrightarrow{g(x)} 8 \xrightarrow{f(x)} 12$; $8 \xrightarrow{g(x)} 10 \xrightarrow{f(x)} 12$ $fog(x) = \{(4,5)(2,5)(6,12)(8,12)\}$ ب) $4 \xrightarrow{f(x)} 5$; $6 \xrightarrow{f(x)} 5$; $8 \xrightarrow{f(x)} 12$; $10 \xrightarrow{f(x)} 12$ $gof(x) = \emptyset$ ج) $4 \xrightarrow{f(x)} 5$; $6 \xrightarrow{f(x)} 5$; $8 \xrightarrow{f(x)} 12$; $10 \xrightarrow{f(x)} 12$ $fof(x) = \emptyset$ د) $4 \xrightarrow{g(x)} 6 \longrightarrow 8$; $2 \xrightarrow{g(x)} 4 \longrightarrow 6$; $6 \xrightarrow{g(x)} 8 \longrightarrow 10$; $8 \xrightarrow{g(x)} 10 \longrightarrow \cancel{\ }$ $gog(x) = \{(4,8)(2,6)(6,10)\}$ (۲)
۵	$f(g(x)) \to 4 \xrightarrow{g(x)} \{\ \} \xrightarrow{f(x)} 3$ $\quad a=4$ $3\ b\ 1$ $2 \longrightarrow 2$ ✓ $(4,5)$ $g(f(x)) \to 4 \longrightarrow \{\ \} \longrightarrow 1$ $\quad b=5$ 5 (۳)

$f(x)=ax+b \Rightarrow f(f(x))=a(ax+b)+b=a^2x+ab+b=4x+3$ $a^2=4 \to a=\pm 2 \begin{cases} +2 \to 4x+2b+b=4x+3b \\ -2 \to 4x-2b+b=4x-b \end{cases}$ $4x+3b=4x+3 \to b=1$; $4x-b=4x+3 \to b=-3$ $f(x)=2x+1$, $f(x)=4x-3$ $\quad f(-1) \Rightarrow \begin{cases} -4+1=-3 \\ -4-3=-7 \end{cases}$ $\quad g(-3)=-9-2=-11$ یا $g(-7)=-15-2=-17$ $2x+3=t \to x=\frac{t-3}{2} \to g(x)=\frac{3x-9}{2}-2$ $f(x)=2x+1 \to f(-1)=-1 \to g(-1)=-8$ / $f(x)=-2x-3 \xrightarrow{f(-1)} -1 \to g(-1)=-8$ 1	۶
$D_{gof}=\{x\in D_f : f(x)\in D_g\}$ $\quad x+\lvert x\rvert \geq 0 \to 2x \geq 0$ $D_{gof}=\left[\begin{cases} x\geq 0 \\ 2x\geq 0 \end{cases} : \sqrt{x+\lvert x\rvert} \neq \{0,2\}\right]$ $\quad D_{gof}=(0,+\infty)-\{2,0\}$ $x\neq 0$ و $x+\lvert x\rvert \neq 4$ $2x\neq 4$ $x\neq 2$ ۰/۵	۷
$D_{(f+g)of}=\{x\in D_f : f(x)\in D_{f+g}\}$ $\quad 1-x^2\geq 0$ $\quad D_f=[-1,1]$ $= \{[-1,1] : f(x)\in\{0,1\}\}$ $\quad f(x)=\{(1,0)(0,1)\}$ $g(x)=\{(1,1)(0,0)\}$ $\quad D_{f+g}=\{1,0\}$ $\sqrt{1-x^2}=0 \to x^2=1 \quad x=\pm 1$ $\{+1,-1,0\}$ $\quad f+g=\{(1,1)(0,1)\}$ $\sqrt{1-x^2}=1 \to x^2=0 \quad x=0$ $\Rightarrow D_{(f+g)of}=[-1,1]$ ۲	۸
الف) $\frac{3x+1}{x-2}=t \to 3x+1=tx-2t \to x(t-3)=2t+1$ $\quad x=\frac{2t+1}{t-3}$ $f(t)=4\left(\frac{2t+1}{t-3}\right)+5 \to f(x)=4\left(\frac{2x+1}{x-3}\right)+5 \to f(x)=\frac{8x+4}{x-3}+5$ ب) $x+\frac{1}{x}=t$ $\quad (x+\frac{1}{x})^3=x^3+\frac{1}{x^3}+\frac{3}{x}+3x \to x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-\frac{3}{x}-3x$ $x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3(\frac{1}{x}+x) \to f(t)=t^3-3t \to f(x)=x^3-3x$ ۲	۹
$f(x)=1 \to x\sqrt{x}=1 \xrightarrow{\sqrt{x}=t} t^2(t)=1 \to t^3=1 \to t=1 \to \sqrt{x}=1 \to x=1$ $f(x)=2\sqrt{2} \to x\sqrt{x}=2\sqrt{2} \to x=2$ $\quad 2-1=1$ ۲	۱۰

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$D_f: x+|x| \geq 0 \begin{cases} x \geq 0 \to 2x \geq 0 \to x \geq 0 \\ x \leq 0 \to 0 \geq 0 \end{cases} \longrightarrow D_f = \mathbb{R}$

$D_g: x^2 - 4x \neq 0 \to x \neq 0, 4$

$\sqrt{x+|x|} \neq 0 \to x > 0$

$\sqrt{x+|x|} \neq 4 \to x \neq 8$

$\longrightarrow D_{g \circ f} = (0, +\infty) - \{8\}$