نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...یازدهم ریاضی A.....

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$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & : x\le 1\\ \sqrt{x^2+1} & : x>1\end{cases} \Rightarrow fof\left(\frac{\pi}{3}\right)=?=\frac{\pi}{4}$$

$$\Rightarrow f\left(f\left(\frac{\pi}{3}\right)\right)=f(\sqrt{3})=\sqrt{\sqrt{3}^2+1}=2$$

$\sqrt{3}>1$

$\frac{\pi}{3}<1 \rightarrow \cot 12\frac{\pi}{12\times 3}=\cot\frac{\pi}{4}=1$

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الف) $fog\left(\frac{\pi}{3}\right)\rightarrow g\left(\frac{\pi}{3}\right)=2\cos^2\left(\frac{\pi}{3}\right)=\frac{1}{2}$

$g(x)=2\cos^2 x$

$f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}} \Rightarrow \frac{1}{x}=t \rightarrow x=\frac{1}{t} : \sqrt{\frac{\frac{2}{t}-1}{\frac{1}{t^2}}}=\sqrt{t^2(2-t)}=\sqrt{2t-t^2}$

$\Rightarrow \sqrt{2\times\frac{1}{2}-\frac{1}{4}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

ب) $fog(\sqrt{2})\rightarrow g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\xrightarrow{\times(1+\sqrt{2})}\frac{2+\sqrt{2}}{-1}=-\sqrt{2}-2$

$g(x)=\frac{x}{1-x}$

$f(x)=[x] \Rightarrow f(-\sqrt{2}-2)=[-\sqrt{2}-2]=-3$

$-3<-\sqrt{2}-2<-2$

۳

$f(x)=\sin x$

$g(x)=x\sqrt{1-x^2}$

$gof\left(\frac{\pi}{3}\right)\rightarrow f\left(\frac{\pi}{3}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\Rightarrow \frac{\sqrt{3}}{2}\times\sqrt{1-\frac{3}{4}}=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}=\frac{1}{2}$

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$f(x)=\{(3,5),(4,5),(5,12),(12,-1)\}$; $g(x)=\{(3,4),(3,2),(5,1),(-1,5)\}$

الف) $fog(x)=\{(3,5),(2,5),(4,12),(1,12)\}$

ب) $gof(x)=\{\}=\emptyset$

ج) $fof(x)=\{\}=\emptyset$

د) $gog(x)=\{(3,1),(2,4),(4,-1)\}$

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$f=\{(2,1),(3,2),(4,5),(1,2)\}$; $g=\{(1,2),(3,1),(a,3),(b,1)\}$

$(3,2)\in fog$; $(3,1)\in gof \rightarrow (a,b)=?$

$\Rightarrow fog=\{(1,1),(3,2),(a,2),(b,2)\}$

$(3,2)\Rightarrow a=3$

$\Rightarrow gof=\{(2,2),(4,1)\}$

$3\rightarrow x\rightarrow 1$

$b=a$

$\Rightarrow (a,b)=(3,3)$

۶

$f\circ f = 4x+3 \Rightarrow a(ax+b)+b = 4x+3 : a^2x+(ab+b) = 4x+3$

$\Rightarrow a^2=4 : a=\pm 2$

$a=2 : 2b+b=3 : b=1$

$a=-2 : -2b+b=3 : b=-3$

$\Rightarrow f(x) = \begin{cases} 2x+1 \\ -2x-3 \end{cases}$

$g(2x+3) = 2x-2$

$\Rightarrow g\circ f(-1) = ?$

$2x+3 = t$

$2x = t-3 \rightarrow x = \frac{t-3}{2} : g(t) = 2\left(\frac{t-3}{2}\right)-2 = \frac{2t-6-4}{2} \Rightarrow g(x) = \frac{2x-10}{2}$

$\Rightarrow g\circ f(-1) = \begin{cases} -1 \Rightarrow -2+1 = -1 \\ -1 \Rightarrow 2-3 = -1 \end{cases}$

$\Rightarrow g(-1) = \frac{2(-1)-10}{2} = \frac{-12}{2} = -6$

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$f(x) = \sqrt{x+|x|} \rightarrow |x|+x \geq 0 \rightarrow |x| \geq -x \Rightarrow$ همیشه $\Rightarrow D_f = \mathbb{R}$

$g(x) = \frac{1}{x^2-4x} \rightarrow x^2-4x \neq 0 : x(x-4) \neq 0 \Rightarrow x \neq 0, 4$

$D_{g\circ f} = ? \Rightarrow g\circ f = \{x \in D_f \mid f(x) \in D_g\} \Rightarrow \{x \in \mathbb{R}, \sqrt{x+|x|} \in (0,+\infty)-\{4\}\}$

$\Rightarrow D_{g\circ f} = (0,+\infty)-\{4\}$

(۱) $\Rightarrow \sqrt{x+|x|} \neq 0 : |x| \neq -x \Rightarrow x > 0$

(۲) $\Rightarrow \sqrt{x+|x|} \neq 4 : 2x+|x|-16 \neq 0 : \begin{cases} 3x \neq 16 & x \geq 0 \\ 0 \neq 16 & x < 0 \end{cases} \checkmark \quad x \neq 8$

$\Rightarrow \mathbb{R}-\{8\} \Rightarrow D_g = (0,+\infty)-\{8\}$

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$f(x) = \sqrt{1-x^2} \rightarrow \begin{array}{c|ccc} x & -1 & & 1 \\ \hline 1-x^2 & 0 & + & 0 \end{array} \Rightarrow D_f = [-1,1]$

$g(x) = \sqrt{x} \rightarrow D_g = [0,+\infty)$

$D_{(f+g)\circ f} = ? \Rightarrow D_{(f+g)\circ f} = \{x \in D_f \mid f(x) \in D_{f+g}\} \Rightarrow \{x \in [-1,1] \mid 0 \leq \sqrt{1-x^2} \leq 1\}$

$D_f \cap D_g = [0,1]$

$\Rightarrow D_{(f+g)\circ f} = [0,1]$

$\sqrt{1-x^2} \leq 1$

$\Rightarrow 1-x^2 \leq 1 \rightarrow -x^2 \leq 0$

$\Rightarrow x^2 \geq 0 \Rightarrow$ همیشه ✓

۹

الف) $f\left(\frac{2x+1}{x-2}\right) = 4x+a : 2x+1 = tx-2t$

$x(2-t) = -2t-1 \rightarrow x = \frac{2t-1}{2-t}$

$\Rightarrow f(t) = \frac{4(2t-1)}{2-t}+a = \frac{8t-4+a-at}{2-t} = \frac{2t+11}{2-t} \Rightarrow f(x) = \frac{2x+11}{2-x}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} : \left(x+\frac{1}{x}\right)^3 = x^3+3x+3/x+\frac{1}{x^3} \Rightarrow x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

$\Rightarrow f(x) = x^3-3x$

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$g(x) = x^{0.5} \Rightarrow$ (۱) $x\sqrt{x} = 1 \rightarrow x^3 = 1 \rightarrow x = 1$

$f(x) = x\sqrt{x}$ (۲) $x\sqrt{x} = 2\sqrt{2} \rightarrow x^3 = 8 \rightarrow x = 2$

$\Rightarrow 2-1 =$ (1)

$g\circ f(x) \rightarrow$ ؟ همیشه