

مسئله: صواب یا غلط؟ اگر غلط، تصحیح کنید. A

$$f(x) = \begin{cases} \cot \frac{\pi x}{\varepsilon} & ; x \leq 1 \\ \sqrt{x-1} & ; x > 1 \end{cases} \rightarrow f\left(\frac{1}{4}\right) = \cot \frac{1/4 \pi}{\varepsilon} = \cot \frac{\pi}{4} = \sqrt{3} \quad (1)$$

$$f(f\left(\frac{1}{4}\right)) = f(\sqrt{3}) = \sqrt{\sqrt{3}-1} = \sqrt{2} \quad (2)$$

الف) $g\left(\frac{\pi}{4}\right) = 2 \cos^2 \frac{\pi}{4} = 2 \times \frac{1}{2} = 1$ (3)

$$\rightarrow f \circ g\left(\frac{\pi}{4}\right) = f(g\left(\frac{\pi}{4}\right)) = f(1) = \sqrt{\frac{4-1}{1}} = \sqrt{3} = \sqrt{1/2} \quad (6)$$

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}$ نتیجه $\rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$ (7)

$$\rightarrow f \circ g(\sqrt{2}) = f(g(\sqrt{2})) = f(-\sqrt{2}-2) = [-\sqrt{2}-2] = [-\sqrt{2}] - 2 = -\sqrt{2} - 2$$

$$[-\sqrt{2}] = -2 \quad (9)$$

$g \circ f\left(\frac{\pi}{4}\right) = g(f\left(\frac{\pi}{4}\right)) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1-\frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$ (11)

$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ (12)

الف) $f \circ g(x) = f(g(x)) = \{(4, 5), (4, 5), (6, 12), (8, 12)\}$ (14)

ب) $g \circ f(x) = g(f(x)) = \emptyset$ (15)

ج) $f \circ f(x) = f(f(x)) = \emptyset$ (16)

د) $g \circ g(x) = g(g(x)) = \{(4, 8), (2, 6), (6, 10)\}$ (17)

$(4, 2) \in f \rightarrow f(g(4)) = 2 \rightarrow g(4) = 3 \Rightarrow a = 3$ (18)

$(4, 1) \in g \circ f \rightarrow g(f(4)) = 1 \rightarrow f(4) = 6, f(2) = 5 \Rightarrow b = 5$ (19)

$f(4) = 3$ من صورت سوال (20)

$f(4) = 5$ من صورت سوال

Genobar

$\rightarrow (a, b) = (4, 5)$

$$(f+g) \circ f \leq f \circ f + g \circ f \leq f(f(n)) + g(f(n)) \leq$$

(1)

$$\sqrt{1-(\sqrt{1-x})^2} + \sqrt{1-x^2} \sim 1-x^2 > 0 \rightarrow x^2 < 1 \rightarrow -1 < x < 1$$

$$\hookrightarrow 1-(\sqrt{1-x^2})^2 > 0 \rightarrow (\sqrt{1-x^2})^2 < 1 \rightarrow -1 < \sqrt{1-x^2} < 1$$

$$\sqrt{1-x^2} < 1 \rightarrow 1-x^2 < 1 \rightarrow x^2 > 0$$

: $\sqrt{1-x^2} < 1$ $\rightarrow$ nice

$$\Rightarrow D(f+g) \text{ of } [-1, 1]$$

$$(الف) f\left(\frac{t+1}{x-2}\right) \leq t+1 \rightarrow \frac{t+1}{x-2} = t \rightarrow t+1-t = t(x-2)$$

(9)

$$t(x-2) = t+1$$

$$x(t-2) \leq t+1 \rightarrow x = \frac{t+1}{t-2}$$

$$*: f(t) \leq \frac{t+1}{t-2} \rightarrow \frac{1t+1}{t-2} \leq \frac{1t+1}{t-2} \rightarrow \frac{1t+1}{t-2} \leq \frac{1t+1}{t-2}$$

$$\Rightarrow f(x) = \frac{1x-1}{x-2}$$

$$(ب) f\left(x + \frac{1}{x}\right) \leq x^2 + \frac{1}{x^2} \rightarrow (x + \frac{1}{x})(x^2 + \frac{1}{x^2} - 1) \leq x^2 + \frac{1}{x^2}$$

$$t = x^2 + \frac{1}{x^2} + 2 = 7$$

$$t = x^2 + \frac{1}{x^2} + 2 = 7$$

$$\Rightarrow f(t) \leq t(t-2) \leq t^2 - 2t \rightarrow f(x) \leq x^2 - 2x$$

$$\text{on } x=0 \rightarrow y=0 \rightarrow g(1) \leq 0, g(\sqrt{e}) \leq 0$$

(10)

$$f(x) = x\sqrt{x}$$

$$\rightarrow g \circ f(x) \leq g(f(x)) \leq 0 \rightarrow f(x) \leq 2\sqrt{2} \rightarrow x \leq 2$$

$$f(x) \leq 1 \rightarrow x \leq 1$$

: $x \leq 1$ 