

14, 70

SUBJECT

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Year: Month: Day:

$f(x) = \begin{cases} \cot \frac{\pi x}{\varepsilon} & ; x \leq 1 \\ \sqrt{x-1} & ; x > 1 \end{cases}$
 $f(f(1/4)) = f(\cot \frac{1/4 \pi}{\varepsilon}) = f(\cot \frac{\pi}{4}) = f(\sqrt{3}) = \sqrt{\sqrt{3}-1} = 2$

$g(\pi/4) = 2 \cos^2 \pi/4 = 2 \times 1/2 = 1/2$

$\rightarrow f \circ g(\pi/4) = f(g(\pi/4)) = f(1/2) = \sqrt{\frac{1/2-1}{2}} = \sqrt{\frac{-1/2}{2}} = \sqrt{-1/4}$

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$

$\rightarrow f \circ g(\sqrt{2}) = f(g(\sqrt{2})) = f(-\sqrt{2}-2) = [-\sqrt{2}-2] = [-\sqrt{2}] - 2 = -\sqrt{2} - 2$

$g \circ f(\pi/4) = g(f(\pi/4)) = g(\sqrt{2}/2) = \sqrt{2}/2 \sqrt{1-1/2} = \sqrt{2}/2 \times \frac{1}{\sqrt{2}} = 1/2$

$f(\pi/4) = \sin \pi/4 = \sqrt{2}/2$

$f \circ g(x) = f(g(x)) = \{(4, 5), (4, 5), (7, 12), (8, 12)\}$

$g \circ f(x) = g(f(x)) = \emptyset$

$f \circ f(x) = f(f(x)) = \emptyset$

$g \circ g(x) = g(g(x)) = \{(4, 8), (2, 6), (6, 10)\}$

$(4, 2) \in f \rightarrow f(g(4)) = 2 \rightarrow g(4) = 3 \Rightarrow a = 3$

$(4, 1) \in g \rightarrow g(f(4)) = 1 \rightarrow f(4) = 3, f(2) = 5 \Rightarrow b = 5$

$f(4) = 3$

$f(4) = 5$

$\rightarrow (a, b) = (4, 5)$

Genobar

$$\text{Suppose } f(n) \leq an + b \rightarrow f(f(n)) \leq f(an + b)$$

(Y)

$$f(f(n)) \leq a(an + b) + b \leq \underbrace{a^2 n + ab + b}_{\leq \epsilon n} \leq \epsilon n$$

(5)

$$a^2 \leq \epsilon \rightarrow a \leq \sqrt{\epsilon}$$

$$a \leq \sqrt{\epsilon} \rightarrow \sqrt{\epsilon}b + b \leq \epsilon b \leq \epsilon \rightarrow b \leq 1 \rightarrow f(n) \leq \sqrt{\epsilon}n + 1$$

$$a \leq \sqrt{\epsilon} \rightarrow -\sqrt{\epsilon}b + b \leq -b \leq \epsilon \rightarrow b \leq -\epsilon \rightarrow f(n) \leq -\sqrt{\epsilon}n - \epsilon$$

$$g(\sqrt{\epsilon}n + 1) \leq \sqrt{\epsilon}n + 1 \rightarrow \sqrt{\epsilon}n + 1 \leq t \rightarrow n \leq \frac{t-1}{\sqrt{\epsilon}} \rightarrow g(t) \leq \sqrt{\epsilon} \left(\frac{t-1}{\sqrt{\epsilon}} \right) + 1 \leq$$

$$g(t) \leq \frac{\sqrt{\epsilon}t - \epsilon - 1}{\sqrt{\epsilon}} \leq \frac{\sqrt{\epsilon}t - 1}{\sqrt{\epsilon}}$$

$$(1): f(-1) \leq \sqrt{\epsilon} + 1 \leq 1$$

$$\rightarrow g \circ f(-1) = g(\sqrt{\epsilon} + 1) \leq g(1) \leq \frac{\sqrt{\epsilon}(-1) - 1}{\sqrt{\epsilon}} \leq -1$$

$$(2): f(-1) \leq \sqrt{\epsilon} - 1 \leq -1$$

$$g \circ f = g(f(n)) : \begin{cases} n \in D_f (1) \\ f \in D_g (2) \end{cases}$$

(V)

$$(1): n \in D_f : n + |n| \geq 0 \rightarrow n \geq 0$$

(1, 00)

$$\begin{cases} n \geq 0 \rightarrow 0 \leq 0 \\ n \leq 0 \rightarrow n - n \geq 0 \rightarrow 0 \leq 0 \end{cases} \rightarrow D_g = \mathbb{R}$$

$$(2): f \in D_g : n - \epsilon n \leq n(n - \epsilon) \neq 0 \rightarrow n \neq 0 : \sqrt{n + |n|} \neq 0 \rightarrow n + |n| \neq 0$$

$$\hookrightarrow n \geq 0$$

$$\sqrt{|n|} + n \neq 0 \rightarrow |n| + n \neq 0 \rightarrow n \neq -1 \rightarrow \mathbb{R} - \{-1\}$$

$$n \rightarrow D_{g \circ f} = \mathbb{R} - \{-1\} \subset (\cdot + \alpha) - \{1\}$$



$$(f+g) \circ f \leq f \circ f + g \circ f \leq f(f(n)) + g(f(n)) \leq$$

(1)

$$\sqrt{1-(\sqrt{1-x})^2} = \sqrt{1-x^2} \sim 1-x^2 > 0 \rightarrow x^2 < 1 \rightarrow -1 < x < 1$$

(2)

$$\hookrightarrow 1-(\sqrt{1-x^2})^2 > 0 \rightarrow (\sqrt{1-x^2})^2 < 1 \rightarrow -1 < \sqrt{1-x^2} < 1$$

$$\sqrt{1-x^2} < 1 \rightarrow 1-x^2 < 1 \rightarrow x^2 > 0$$

المجال هو $[-1, 1]$

منه

$$\Rightarrow D(f+g) \text{ of } [-1, 1]$$

$$\text{الف) } f\left(\frac{t+1}{x-2}\right) \leq t+1 \rightarrow \frac{t+1}{x-2} = t \rightarrow t+1 = t(x-2)$$

(3)

$$t+1 = tx - 2t$$

$$x(t-2) \leq t+1 \rightarrow x = \frac{t+1}{t-2}$$

$$*: f(t) \leq \frac{t+1}{t-2} \rightarrow \frac{1t+1}{t-2} \leq \frac{1t+1}{t-2} \leq \frac{1t+1}{t-2}$$

(4)

$$\Rightarrow f(x) = \frac{1x-1}{x-2}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) \leq x^2 + \frac{1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right) \leq x^2 + \frac{1}{x^2}$$

$$t = x^2 + \frac{1}{x^2} + 2 = t$$

$$t = x^2 + \frac{1}{x^2} + 2 = t$$

$$\Rightarrow f(t) \leq t(t-2) \leq t^2 - 2t \rightarrow f(x) \leq x^2 - 2x$$

$$\text{منه } x \geq 0 \rightarrow y = 0 \rightarrow g(1) \leq 0, g(\sqrt{e}) = 0$$

(5)

$$f(x) = x\sqrt{x}$$

$$\rightarrow g \circ f(x) \leq g(f(x)) \leq 0 \rightarrow f(x) \leq 2\sqrt{x} \rightarrow x \leq 2$$

$$f(x) \leq 1 \rightarrow x \leq 1$$

منه $x \leq 1$ 