۲۰

ماهف میرحقتانی ←

$f(f(\frac{\pi}{3}))$

$\cot \frac{\pi}{2} \times \frac{\pi}{3} = \cot \frac{\pi}{4} = \sqrt{3}$

$f(\sqrt{3}) \to \sqrt{3+1} = 2$

$f(g(\frac{\pi}{3})) \to g(\frac{\pi}{3}) = 3 \times \frac{1}{3^2} = \frac{1}{3}$

$f(\frac{1}{3}) = f(x) = \sqrt{\frac{x-1}{x}} = \frac{\sqrt{3}}{2}$

$f(g(\sqrt{2})) \to g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} =$

$-(\sqrt{2}+2) = -\sqrt{2}-2 \to f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -3$

$g(f(\frac{\pi}{12})) \to f(\frac{\pi}{6}) = \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} \to g(\frac{\sqrt{3}}{2}) =$

$\frac{\sqrt{3}}{2} \sqrt{\frac{1-\frac{3}{4}}{\frac{3}{4}}} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{3} = \frac{1}{2}$

$f(g(x)) = \{(1,3),(2,3),(4,12),(5,12)\}$ ← ۲ (۳)

$g(f(x)) = \{\}$ $\quad f(f(x)) = \{\}$

$g(g(x)) = \{(1,5),(2,4),(4,10)\}$

$(1,2) \in f(g(x))$ $\quad g(x) = 2$ $\quad 1$

$\rightarrow a = 2$

$\rightarrow (a,b) = (2,3)$

$(1,1) \in g(f(x)) \rightarrow$ $\quad 3$

$g(3) = 1 \rightarrow b = 3$ ← ۳ (۲)

$f(f(x)) = 2x + 3$ ← ۹ (۳)

$g(2x+3) = 3x - 2$

$g(f(-1)) = ?$

$g(2x+3) \Rightarrow 2x+3 = 1 \rightarrow x = \frac{\pm 2}{2}$

t ←

$f(\underbrace{f(x)}_{-1}) = 4x - 1 + 3 = -1$

$g(-1) \longrightarrow 4m + 3 = -1 \rightarrow 4m = -4 \rightarrow m = -2$

$g(-1) = 3x - 2 - 2 = \boxed{-8}$

۷ ←

$f(x) = \sqrt{x+|x|}$

$g(x) = \dfrac{1}{x^2-4x} \rightarrow D_{g(f(x))} = ?$

$$\rightarrow \frac{1}{(x+|x|)-4\sqrt{x+|x|}} \rightarrow$$

همیشه + یا صفر ← همیشه + یا صفر

$$x+|x|-4\sqrt{x+|x|} \neq 0 \rightarrow$$

$R^-, 0 \rightarrow x - x \neq 4\sqrt{x-x} \rightarrow$ چون طرفین همیشه ۰

پس R^- ها متغیر، صفر نمی‌تونن باشن

$$R^+ \rightarrow 2x \neq 4\sqrt{2x} \rightarrow x \neq 2\sqrt{2x} \rightarrow$$

$$\sqrt{x}(\sqrt{x}-2\sqrt{2}) \neq 0 \rightarrow x \neq 8 \rightarrow$$

$$\boxed{D_{gof} = (0,+\infty) - \{8\}}$$

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۸ ←

$f(x) = \sqrt{1-x^2}$ $\quad g(x) = \sqrt{x}$

$(f+g)\circ f$

$\sqrt{1-(1-x^2)} + \sqrt{1-x^2} = \pm x + \sqrt{1-x^2}$

$-1 \quad 1$

$- \quad + \quad -$

$D_{(f+g)\circ f} = [-1,1]$

$f\left(\frac{3x+1}{x-2}\right) = 4x+3$

$\frac{3x+1}{x-2} = t \rightarrow 3x+1 = tx-2t \rightarrow x(3-t) = -2t-1 \rightarrow$

$x = \frac{2t+1}{t-3} \rightarrow f(t) = \frac{8t+4+3t-15}{t-3} =$

$f(t) = \frac{11t-11}{t-3}$

$f(x) = \frac{11x-11}{x-3} \quad x \neq 3$

$f(x+\frac{1}{x}) = (x^2+\frac{1}{x^2})$ Q-9

$\hookrightarrow t$

$t^2 = x^2+\frac{1}{x^2}+2x+\frac{2}{x} \rightarrow 2(x+\frac{1}{x})$

$t^2-2t = x^2+\frac{1}{x^2} \longrightarrow f(t) = t^2-2t \rightarrow$

$\boxed{f(x) = x^2-2x}$

$g(x) = (x-1)(x-2\sqrt{2})$

$f(x) = x\sqrt{x} \rightarrow g(f(x)) = (x\sqrt{x}-1)(x\sqrt{x}-2\sqrt{2})$

$g(f(x)) = (x\sqrt{x}-1)(x\sqrt{x}-2\sqrt{2}) \rightarrow x = 1, 2$

$\rightarrow |2-1| = 1$
