

$$f(x) = \begin{cases} \cot \frac{\pi}{x} & : x < 1 \\ \sqrt{x^2 + 1} & : x > 1 \end{cases} \quad f \circ f \left(\frac{1}{x} \right) \Rightarrow f \left(\frac{1}{x} \right) = \frac{\pi \times x}{x \times x} \cot = \cot \frac{\pi}{x} = \sqrt{x}$$

$$f \circ g \left(\frac{\pi}{x} \right) \Rightarrow f \left(g \left(\frac{\pi}{x} \right) \right) \Rightarrow g \left(\frac{\pi}{x} \right) = x \times \cos^2 \frac{\pi}{x} = x \times \frac{1}{x} = \frac{1}{x}$$

$$f \left(\frac{1}{x} \right) = \sqrt{\frac{x^2 - 1}{x^2}} \Rightarrow x = 2 \Rightarrow \sqrt{\frac{x-1}{x}} = \frac{\sqrt{x}}{x} \quad f \circ g \left(\frac{\pi}{x} \right) = \left[\frac{\sqrt{x}}{x} \right]$$

$$f \circ g(\sqrt{x}) = f(g(\sqrt{x})) \Rightarrow g(\sqrt{x}) = \frac{\sqrt{x}}{1 - \sqrt{x}} \Rightarrow f \left(\frac{\sqrt{x}}{1 - \sqrt{x}} \right) = \left[\frac{\sqrt{x}}{1 - \sqrt{x}} \right]$$

$$\Rightarrow \frac{\sqrt{x}}{1 - \sqrt{x}} \times \frac{1 + \sqrt{x}}{1 + \sqrt{x}} = \frac{\sqrt{x}(1 + \sqrt{x})}{1 - x} = \frac{x + \sqrt{x}}{-1} = -x - \sqrt{x} \Rightarrow [-(x + \sqrt{x})] \Rightarrow$$

$$\sqrt{x} = 1/2 \rightarrow [-(x + \sqrt{x})] = [-x/2] = \boxed{-1/4}$$

$$g \circ f = g(f(x)) \Rightarrow x = \frac{\pi}{x} \Rightarrow f(x) = \sin x \quad f \left(\frac{\pi}{x} \right) = \sin \frac{\pi}{x} = \frac{\sqrt{x}}{x}$$

$$g \left(\frac{\sqrt{x}}{x} \right) = \frac{\sqrt{x}}{x} \sqrt{1 - \left(\frac{\sqrt{x}}{x} \right)^2} = \frac{\sqrt{x}}{x} \times \frac{\sqrt{x}}{x} = \frac{x}{x} = \frac{1}{x}$$

$$\text{الف)} f \circ g(x) = f(g(x)) \Rightarrow x \rightarrow y \rightarrow a/2 \rightarrow x \rightarrow a/4 \rightarrow a \rightarrow 12/a \rightarrow 10 \rightarrow 12$$

$$= \{(x, a)(x, a)(y, 12)(a, 12)\}$$

$$\text{ب)} g \circ f(x) = g(f(x)) \Rightarrow x \rightarrow a \rightarrow x/4 \rightarrow a \rightarrow x/8 \rightarrow 12 \rightarrow x/10 \Rightarrow 12 \rightarrow x \quad \emptyset$$

$$\text{ج)} f \circ f = f(f(x)) \Rightarrow x \rightarrow a \rightarrow x/4 \rightarrow a \rightarrow x/8 \rightarrow 12 \rightarrow x/10 \rightarrow 12 \rightarrow x \quad \emptyset$$

$$\text{د)} g \circ g = g(g(x)) \Rightarrow x \rightarrow y \rightarrow a/2 \rightarrow x \rightarrow y/4 \rightarrow a \rightarrow 10/a \rightarrow 10 \rightarrow x/$$

$$= \{(x, a)(x, y)(y, 10)\}$$

$$f(g(x)) \ni (x, x) \Rightarrow x \rightarrow \frac{x}{x} \rightarrow x \rightarrow x = x \Rightarrow a = x$$

$$g(f(x)) \ni (x, 1) \Rightarrow x \rightarrow x \rightarrow 1 \quad b = a \quad (a, b) \Rightarrow (x, a)$$

$$f(f(x)) = x + 2 \Rightarrow f(f(x)) = f(x) + 2 \quad f(x) < x + 1$$

$$g(x + 2) = x - 2 \Rightarrow x + 2 = x \Rightarrow x = \frac{x - 2}{x} \quad \text{① } g(x) = \frac{x - 1}{x}$$

$$g(x) = \frac{x - 1}{x} - 2 \Rightarrow g(x) = \frac{x - 1 - 2x}{x} \quad \text{② } g \circ f(-1) = g(f(-1)) =$$

$$g(x \times (-1) + 1) = g(-1) = \frac{x \times (-1) - 1}{x} = \frac{-x - 1}{x} = \boxed{-1 - \frac{1}{x}}$$

$$g(f(x)) \Rightarrow D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} = \{x \in \mathbb{R}, (0, +\infty) - \{1\}\}$$

$$D_f \Rightarrow x + |x| \geq 0 \quad \sqrt{(x + |x|)^2} - x \sqrt{(x + |x|)} \neq 0 \Rightarrow \text{مطلوبه}$$

$$\Rightarrow (x + |x|) - x \sqrt{(x + |x|)} = 0 \quad \begin{cases} \mathbb{R}^+ \times \\ \mathbb{R}^+ \Rightarrow x - x \sqrt{x} = 0 \\ 0 \times \Rightarrow \text{if } x = 0 \end{cases}$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$$

$$D(f+g) \text{ of } f = \{ x \in Df, Df \in D(g+f) \} = \{ -1 \leq x \leq 1, x \in [0,1] \}$$

$$D(g+f) \begin{cases} Df \rightarrow 1-x^2 \geq 0 \quad -1 \leq x \leq 1 \\ Dg \rightarrow [0, +\infty) \end{cases} \cap [0,1]$$

$$\Rightarrow x \in [0,1]$$

جواب: $[0,1]$

الف) $f\left(\frac{r+1}{n-r}\right) = rn + a \quad t = \frac{r+1}{n-r} \Rightarrow t(n-r) = r+1$

$$n(t-r) = r+1 \Rightarrow n = \frac{r+1}{t-r} \quad \leftarrow t(n-r) = r+1$$

$$f(t) = r \times \left(\frac{r+1}{t-r}\right) + a \Rightarrow f(t) = \frac{rt+r}{t-r} + a \quad f(n) = \frac{rn+r}{n-r} + a$$

ب) $f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \Rightarrow \left(n + \frac{1}{n}\right)^r = n^r + \frac{1}{n^r} + rn^r \frac{1}{n}$

$$+ rn^r \frac{1}{n^r} = n^r + \frac{1}{n^r} + rn + r \frac{1}{n} = n^r + \frac{1}{n^r} + r\left(n + \frac{1}{n}\right)$$

$$f(n) = n^r - r(n)$$

$$g(\sqrt{2}) = 0, \quad g(1) = 0 \quad f(n) = n\sqrt{n}$$

$$g(f(n)) = 0 \quad \begin{cases} f(n) = \sqrt{2} \Rightarrow n = 2 \\ f(n) = 1 \Rightarrow n = 1 \end{cases}$$

مروطه ها: $y=0$

$$2-1 = \boxed{1} \quad \text{اختلاف}$$