

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & : x < 1 \\ \sqrt{x^2 + 1} & : x > 1 \end{cases} \quad f \circ f \left(\frac{1}{4} \right) \Rightarrow f \left(\frac{1}{4} \right) = \frac{\pi \times \frac{1}{4}}{1 \times 4} \cot = \cot \frac{\pi}{4} = \sqrt{4}$$

(۳)

$$f \circ g \left(\frac{\pi}{4} \right) \Rightarrow f \left(g \left(\frac{\pi}{4} \right) \right) \Rightarrow g \left(\frac{\pi}{4} \right) = 2 \times \cos^2 \frac{\pi}{4} = 2 \times \frac{1}{4} = \frac{1}{2}$$

(الف) ۲

$$f \left(\frac{1}{2} \right) = \sqrt{\frac{2 \times \frac{1}{2} - 1}{\frac{1}{2}}} \Rightarrow n = 2 \Rightarrow \sqrt{\frac{1 - 1}{2}} = \frac{\sqrt{2}}{2} \quad f \circ g \left(\frac{\pi}{4} \right) = \left[\frac{\sqrt{2}}{2} \right]$$

(۳)

$$f \circ g (\sqrt{2}) = f (g (\sqrt{2})) \Rightarrow g (\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \Rightarrow f \left(\frac{\sqrt{2}}{1 - \sqrt{2}} \right) = \left[\frac{\sqrt{2}}{1 - \sqrt{2}} \right]$$

(ب)

$$\Rightarrow \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{\sqrt{2}(1 + \sqrt{2})}{1 - 2} = \frac{2 + \sqrt{2}}{-1} = -2 - \sqrt{2} \Rightarrow [-(2 + \sqrt{2})] \Rightarrow$$

$$\sqrt{2} \sim 1.4 \rightarrow [-(2 + \sqrt{2})] \sim [-3.4] = [-4]$$

$$g \circ f = g (f(x)) \Rightarrow x = \frac{\pi}{4} \Rightarrow f(x) = \sin x \quad f \left(\frac{\pi}{4} \right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

(۳)

$$g \left(\frac{\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2} \right)^2} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$$\text{الف) } f \circ g (x) = f (g(x)) \Rightarrow x \rightarrow y \rightarrow a/2 \rightarrow x \rightarrow a/4 \rightarrow a \rightarrow 12/a \rightarrow 10 \rightarrow 12$$

(۳)

$$= \{ (x, a)(x, a)(y, 12)(a, 12) \}$$

$$\text{ب) } g \circ f (x) = g (f(x)) \Rightarrow x \rightarrow a \rightarrow x/4 \rightarrow a \rightarrow x/a \rightarrow 12 \rightarrow x/10 \Rightarrow 12 \rightarrow x \quad \emptyset$$

$$\text{ج) } f \circ f = f (f(x)) \Rightarrow x \rightarrow a \rightarrow x/4 \rightarrow a \rightarrow x/a \rightarrow 12 \rightarrow x/10 \rightarrow 12 \rightarrow x \quad \emptyset$$

$$\text{د) } g \circ g = g (g(x)) \Rightarrow x \rightarrow y \rightarrow a/2 \rightarrow x \rightarrow y/4 \rightarrow a \rightarrow 10/a \rightarrow 10 \rightarrow x/$$

$$= \{ (x, a)(x, a)(y, 10) \}$$

$$f(g(x)) \ni (x, 2) \Rightarrow x \rightarrow \frac{x}{2} \rightarrow 2 \rightarrow x = 2 \Rightarrow a = x$$

(۳)

$$g(f(x)) \ni (x, 1) \Rightarrow x \rightarrow a \rightarrow 1 \quad b = a \quad (a, b) \Rightarrow (x, a)$$

$$f(f(x)) = 2x + 2 \Rightarrow f(f(x)) = f(x) + 2 \quad f(x) < 2x + 1$$

$$g(2x + 2) = 2x - 2 \Rightarrow 2x + 2 = x \Rightarrow x = \frac{x - 2}{2} \quad \text{① } g(x) = \frac{x - 12}{2}$$

(۳)

$$g(x) = \frac{x - 9}{2} - 2 \Rightarrow g(x) = \frac{x - 13}{2} \quad \text{① } g \circ f (-1) = g (f(-1)) =$$

$$g(2 \times (-1) + 1) = g(-1) = \frac{2 \times (-1) - 13}{2} = \frac{-15}{2} = [-8]$$

$$g(f(x)) \Rightarrow D_{g \circ f} = \{ x \in D_f : f(x) \in D_g \} = \{ x \in \mathbb{R}, (0, +\infty) - \{1\} \}$$

$$D_f \Rightarrow x + |x| \geq 0 \quad \sqrt{(x + |x|)^2} - x \sqrt{(x + |x|)} \neq 0 \Rightarrow \text{مطابق } g(x) \quad \mathbb{R}^+ - \{1\}$$

(۳)

$$\Rightarrow (x + |x|) - x \sqrt{(x + |x|)} = 0 \quad \begin{cases} \mathbb{R}^- \times \\ \mathbb{R}^+ \Rightarrow 2x - x \sqrt{2x} \approx 0 \\ 0 \times \Rightarrow \text{if } x = 1 \end{cases}$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$$

$$D(f+g) \text{ of } f = \{ x \in Df, Df \in D(g+f) \} = \{ -1 \leq x \leq 1, x \in [0,1] \}$$

$$D(g+f) \begin{cases} Df \rightarrow 1-x^2 \geq 0 \quad -1 \leq x \leq 1 \\ Dg \rightarrow [0, +\infty) \end{cases} \cap [0,1]$$

$$\Rightarrow x \in [0,1] \quad \text{جواب نهایی}$$

$$\text{الف) } f\left(\frac{r+1}{n-r}\right) = rn + a \quad t = \frac{r+1}{n-r} \Rightarrow t(n-r) = rn + 1$$

$$n(t-r) = rt + 1 \Rightarrow n = \frac{rt+1}{t-r} \quad \leftarrow t(n-r) = rt+1$$

$$f(t) = r \times \left(\frac{rt+1}{t-r}\right) + a \Rightarrow f(t) = \frac{rt+rt^2}{t-r} + a \quad f(n) = \frac{rn+r^2}{n-r} + a$$

$$\text{ب) } f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \Rightarrow \left(n + \frac{1}{n}\right)^r = n^r + \frac{1}{n^r} + rn^r \frac{1}{n} + rn \frac{1}{n^r} = n^r + \frac{1}{n^r} + r(n + \frac{1}{n})$$

$$f(n) = n^r - r(n)$$

$$g(\sqrt{2}) = 0, \quad g(1) = 0 \quad f(n) = n\sqrt{n}$$

$$g(f(n)) = 0 \quad \begin{cases} f(n) = \sqrt{2} \Rightarrow n = 2 \\ f(n) = 1 \Rightarrow n = 1 \end{cases}$$

مجموعه ها را قطع می کنیم : $y=0$

$$2-1 = \boxed{1} \quad \text{اختلاف}$$