

سوال 1) در تابع $f(x) = \begin{cases} \cot \frac{\pi x}{2} ; x < 1 \\ \sqrt{x^2 + 1} ; x > 1 \end{cases}$ مقدار $f \circ f(\frac{1}{3})$ ؟

2) $\sqrt{x^2 + 1} ; x > 1$

$\cot \frac{\pi}{2} \times \frac{1}{3} = \cot \frac{\pi}{6} = \sqrt{3} \rightarrow \sqrt{(\sqrt{3})^2 + 1} = 2$

سوال 2) الف) $f(x) = \sqrt{\frac{x-1}{x^2}}$ و $g(x) = 2 \cos^2 x$ مقدار $f \circ g(\frac{\pi}{4})$ ؟

3) $2 \times (\frac{1}{2})^2 = \frac{1}{2} = g(\frac{\pi}{4}) \Rightarrow \frac{1}{x} \rightarrow \frac{1}{2} \Rightarrow 2 \Rightarrow f(2) = \frac{\sqrt{3}}{2}$

ب) اگر $f(x) = [x]$ و $g(x) = \frac{x}{1-x}$ مقدار $f \circ g(\sqrt{2})$ ؟

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2 \in [-\sqrt{2}-2, -\sqrt{2}-2] = -4$

سوال 3) $f(x) = \sin^{-1} x$ و $g(x) = n\sqrt{1-x^2}$ مقدار $f \circ g(\frac{1}{2})$ ؟

4) $f(\frac{1}{2}) = \frac{\sqrt{3}}{2}$ و $g(\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2} \sqrt{1-\frac{3}{4}} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{1}}{2} = \frac{\sqrt{3}}{4}$

سوال 4) $f(x) = \{(x, 0), (y, 0), (1, 12), (10, 12)\}$ و $g(x) = \{(x, 4), (y, 4), (4, 1), (1, 12)\}$

الف) $f \circ g(x) \rightarrow x \rightarrow 4 \rightarrow 0 / 2 \rightarrow 4 \rightarrow 0 / 4 \rightarrow 1 \rightarrow 12 / 10 \rightarrow 12$

$= \{(0, 0), (2, 0), (4, 12), (10, 12)\}$

ب) $f \circ g(x) \rightarrow x \rightarrow 0 \rightarrow x / 4 \rightarrow 0 \rightarrow x / 1 \rightarrow 12 \rightarrow x / 10 \rightarrow 12 \rightarrow x$

$= \{\emptyset\}$

ج) $f \circ f(x) \rightarrow x \rightarrow 0 \rightarrow x / 4 \rightarrow 0 \rightarrow x / 1 \rightarrow 12 \rightarrow x / 10 \rightarrow 12 \rightarrow x$

$= \{\emptyset\}$

د) $f \circ g(x) \rightarrow x \rightarrow 4 \rightarrow 1 / 2 \rightarrow 4 \rightarrow 0 / 4 \rightarrow 1 \rightarrow 10 / 1 \rightarrow 10 \rightarrow x$

$= \{(4, 1), (2, 0), (4, 10)\}$

Arman

1. $(\varepsilon, 2) \in f \circ g$ $g = \{(1, 2), (3, 1), (a, 3), (b, 3)\}$ $f = \{(2, 1), (3, 2), (c, d), (d, 1)\}$ (5)
 2. $(1, \varepsilon) \in f$ $\Rightarrow$ (a, b) در f نیست. (5)

$$f(g(\varepsilon)) = 2 \Rightarrow g(\varepsilon) = 3 \Rightarrow a = \varepsilon$$

$$g(f(\varepsilon)) = 1 \rightarrow f(\varepsilon) = 2 \Rightarrow b = d$$

$$(a, b) = (\varepsilon, d)$$

$$g(2n+2) = 2n-2, f(f(n)) = 2n+2 \quad \text{سوال (40)}$$

$$a(an+b)+b = a^2n+ab+b = 2n+2 \quad f(-1) = -1 \quad \text{سوال (5)}$$

$$\Rightarrow a^2 = 2 \rightarrow a = 2 \rightarrow 2b+b = 2 \rightarrow b = 1 \xrightarrow{(1)} f(n) = 2n+1$$

$$\rightarrow a = -2 \rightarrow -2b+b = 2 \rightarrow b = -2 \xrightarrow{(2)} f(n) = -2n-2$$

$$2n+2 = t \rightarrow n = \frac{t-2}{2} \Rightarrow g(t) = 2\left(\frac{t-2}{2}\right) - 2 = \frac{t-4}{2} - 2 = \frac{t-8}{2}$$

$$g \circ f(-1) = g\left(\frac{f(-1)}{-1}\right) \Rightarrow g(-1) = \frac{2(-1)-4}{2} = -1$$

$$g(1) = 0, g(2\sqrt{2}) = 0 \rightarrow f(n) = n\sqrt{2} \quad \text{سوال (10)}$$

$$g \circ f(n) = g(f(n)) = 0 \rightarrow f(n) = 1 \Rightarrow n = 1$$

$$f(n) = 2\sqrt{2} \Rightarrow n = 2$$

$$2-1=1$$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50

$$g \circ f \text{ is, } g(n) = \frac{1}{n^2 - \varepsilon n}, \quad f(n) = \sqrt{n + |n|} \quad (\text{V.O.L.})$$

$$\begin{cases} n \in D_f \rightarrow D_f \Rightarrow n + |n| \geq 0 \rightarrow n \geq -|n| \rightarrow D_f = \mathbb{R} \\ f(n) \in D_g \end{cases}$$

$$\hookrightarrow n^2 - \varepsilon n \neq 0 \rightarrow D_f = \mathbb{R} - \{0\}$$

$$\begin{aligned} n + |n| \neq 0 &\rightarrow n \neq -|n| \rightarrow n > 0 \\ n + |n| \neq 1 &\rightarrow n \neq 1 \end{aligned} \quad \left. \vphantom{\begin{aligned} n + |n| \neq 0 \\ n + |n| \neq 1 \end{aligned}} \right\} D_{g \circ f} = (0, +\infty) - \{1\}$$

10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50

$$(f+g) \circ f \text{ is, } g(n) = \sqrt{n}, \quad f(n) = \sqrt{1-n^2} \quad (\text{V.O.L.})$$

$$\sqrt{1-n^2} + \sqrt{n} \quad \sqrt{1-n^2} \quad \sqrt{1-1+n^2} = \sqrt{1-n^2} = \sqrt{n^2}$$

$$\sqrt{1-n^2} = n + \sqrt{1-n^2} \rightarrow 1-n^2 = -n^2 \quad (\text{V.O.L.})$$

$$\Rightarrow -1/n^2$$

$$\Rightarrow D_f = [-1, 1]$$

(V.O.L.)

18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50

$$f\left(\frac{t^{1/3}+1}{t-1}\right) = \varepsilon t + \delta \rightarrow \frac{t^{1/3}+1}{t-1} \cdot t \rightarrow t^{1/3} - 1 = t^{1/3} + 1$$

$$t^{1/3} - t^{1/3} = -1$$

$$n(t^{1/3} - 1) = -1$$

$$n = \frac{t^{1/3} + 1}{t-1} \quad (\text{V.O.L.})$$

$$\rightarrow f(n) = \frac{13n-11}{t-1}$$

$$f\left(n + \frac{1}{n}\right) = n^3 + \frac{1}{n^3}$$

$$\left(n + \frac{1}{n}\right)^3 = n^3 + \frac{1}{n^3} + 3 \times n \times \frac{1}{n} \left(n + \frac{1}{n}\right) \Rightarrow n^3 + \frac{1}{n^3} = t^{1/3} - 1$$

$$\Rightarrow f(t) = t^{1/3} - 1 \rightarrow f(n) = n^3 - 1$$

Arman