

$$f\left(\frac{\pi}{4}\right) \xrightarrow{x \leq 1} \cos \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$f(\sqrt{2}) \xrightarrow{x > 1} \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3} = \underline{2, -2}$$

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$$g\left(\frac{\pi}{4}\right) = 2 \cos \frac{\pi}{4} = 2 \left(\frac{\sqrt{2}}{2}\right) = \sqrt{2} \rightarrow f\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{2(2)-1}{2}} = \sqrt{\frac{3}{2}} = \pm \frac{\sqrt{6}}{2}$$

$$b) g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$$

$$f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -4$$

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$$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

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$$a) f \circ g(x) = \{(1, 2), (2, 2), (4, 12), (8, 12)\}$$

$$b) g \circ f(x) = \emptyset$$

$$c) f \circ f(x) = \emptyset$$

$$d) g \circ g(x) = \{(1, 1), (2, 2), (4, 10)\}$$

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$$f \circ g = \{(1, 1), (3, 7), (a, 2), (b, 7)\} \text{ و } (1, 2) \in f \circ g \Rightarrow a = 4$$

$$g \circ f = \{(2, 2), (1, 1)\}$$

$$f(x) = (1, 2), g(x) = (b, 1), (1, 1) \in g \circ f \Rightarrow b = 2 \Rightarrow (a, b) = (4, 2)$$

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$f(x) \rightarrow a(ax+b) + b \leq x + 2 \rightarrow a^2x + ab + b \leq x + 2 \Rightarrow a^2 = 1, a = -1, 1, ab + b = 2$ $\Rightarrow b = -1, 1 \Rightarrow f(x) = -x - 1, 1 - x + 1$ $f(-1) = -(-1) - 1 = 0, 1 - (-1) + 1 = 2 \Rightarrow f(-1) = 1$ $g(-1) \Rightarrow x + 2 = -1, x = -3, x = -2 \Rightarrow 3(-2) - 2 = -8$	6
$D_{g \circ f} = \{x \in D_f, f(x) \in D_g\}$ $D_f = x + x \geq 0 \Rightarrow x \geq 0 = [0, +\infty), D_g = x^2 - 4x \neq 0, x(x-4) \neq 0 \Rightarrow D_g = \mathbb{R} - \{0, 4\}$ $D_{g \circ f} = \{x \in [0, +\infty) \mid \sqrt{x+ x } \in \mathbb{R} - \{0, 4\}\} \Rightarrow D_{g \circ f} = (0, +\infty) - \{4\}$ درمانشیت $\sqrt{4} = 2 \leftarrow x \neq 4$ ، درمانشیت $\{0\}$ چون $x \geq 0$	7
$D_g = x \geq 0, D_f = 1 - x^2 \geq 0 \Rightarrow -1 \leq x \leq 1, \Rightarrow D_{(f+g)} = [0, +\infty) \cap [-1, 1] = [0, 1]$ $D_{(f+g) \circ f} = \{x \in D_f, f(x) \in D_{(f+g)}\} \rightarrow \{x \in [-1, 1], 0 \leq \sqrt{1-x^2} \leq 1\}$ $0 \leq 1 - x^2 \leq 1 \rightarrow -1 \leq -x^2 \leq 0 \xrightarrow{x \geq 0} 1 \geq x^2 \geq 0 \Rightarrow 0 \leq x \leq 1$ $D_{(f+g) \circ f} = [-1, 1] \cap [0, 1] = [0, 1]$	8
الف) $\frac{3x+1}{x-2} = z \rightarrow xz - 2z = 3x+1, x(z-3) = 1+2z, x = \frac{1+2z}{z-3} \Rightarrow f(x) = f\left(\frac{1+2z}{z-3}\right) + 5$ $\frac{f+12+5(z-3)}{z-3} = \frac{f+12+5z-15}{z-3} = \frac{13z-11}{z-3} \Rightarrow f(x) = \frac{13x-11}{x-3}$ ب) $(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2 \rightarrow x^2 + \frac{1}{x^2} + 2 = 3 \Rightarrow x^2 + \frac{1}{x^2} = 1 \Rightarrow f(x) = x^2 - 3x$	9
$g(\sqrt{x}) = 0, g(1) = 0$ $g \circ f(x) = 0 \Rightarrow f(x) = x\sqrt{x}, f(x) = 1$ $f(x) = x\sqrt{x} = x\sqrt{x} \Rightarrow x = 1, f(x) = x\sqrt{x} = 1 \Rightarrow x = 1$ $ x-1 = 2-1 = 1$	10