۱)
$f(x)=\sqrt{1-x^2} \qquad g=\{(-1,1),(0,4),(2,0),(1,2)\}$

$D_f: 1-x^2\geq 0 \to x^2\leq 1 \to |x|\leq 1 \to -1\leq x\leq 1 \to D_f\cap D_g=\{-1,0,1\}$

$D_g=\{-1,0,2,1\}$

$(2g-3f)(1)=2g(1)-3f(1)=2(2)-3(0)=4$

$(2g-3f)(0)=2g(0)-3f(0)=2(4)-3(1)=8-3=5 \to 4+5+2=11$

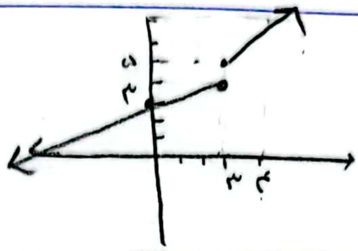
$(2g-3f)(-1)=2g(-1)-3f(-1)=2(1)-3(0)=2$

۲)
$f(x)=2x-1$

x	3	4
y	5	7

$g(x)=\frac{1}{3}x+3$

x	0	3
y	3	4



$R=(-\infty,4]\cup[5,+\infty)$

۳)
$y=-\frac{x^2}{2}+x+3 \to$

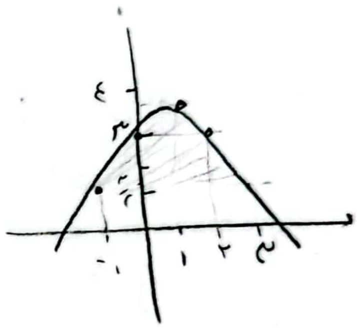
$-2(\frac{3}{2}=-\frac{x^2}{2}+x+3) \to -3=x^2-2x-6 \to x^2-2x-3=0$

$\to (x-3)(x+1)=0 \to \begin{cases}x=3\\x=-1\end{cases}$

$x_s=\frac{-b}{2a}=1 ,\ y_s=f(1)=\frac{7}{2}$

x	-1	0	1	2	3
y	3/2	3	7/2	3	3/2

جواب: $(a,b)=(-1,3)$

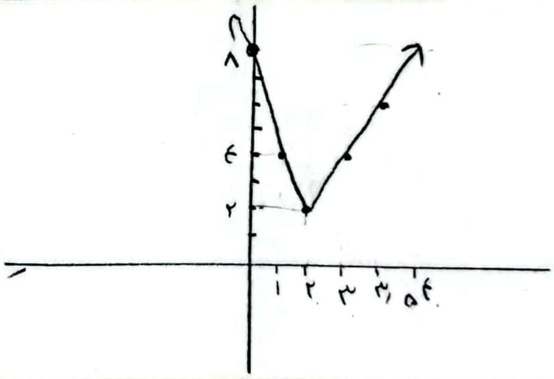


۴)
$y=|x-1|+|x-3|+|2x-4|$

$x=1,3,2$

x	0	1	2	3	3.5	4
y	8	4	2	4	6	8

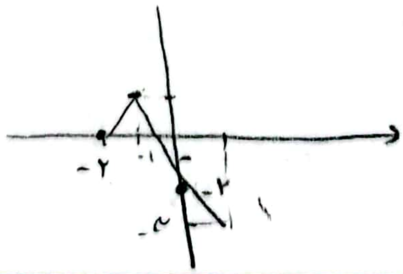
کمترین مقدار: $y=2$



۵)
$y=|x|-2|x+1|$

ریشه‌ها: $x=0 ,\ x=-1$

x	-2	-1	0	1
y	0	1	-2	-3



$R=(-\infty,1]$

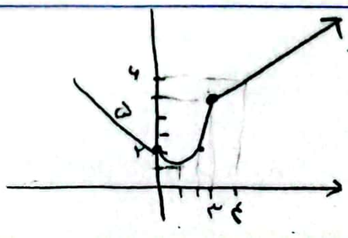
۶)
$y=\frac{x^2+5x+m}{x+1} \qquad x+1=0\to x=-1 \qquad (-1)^2+5(-1)+m=0 \to \underbrace{1-5}_{-4}+m=0 \to$

$\boxed{m=4}$

۷)
$f(x)=\begin{cases}x+2 & x\geq 3\\ x^2-2x+1+1=(x-1)^2+1 & 0\leq x<3\\ |x|+2 & x<0\end{cases}$

x	3	4
y	5	6

x	0	1	2	3
y	2	1	2	5



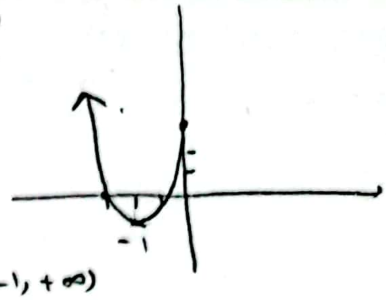
$R_f=[1,+\infty)$

۱۸

$$f(x)=\begin{cases} x^2+4x+3+1-1 \\ (x+2)^2-1 & x\leq 0 \\ [2x]-2x & x>0 \end{cases}$$

$-1<[2x]-2x\leq 0$

x	-3	-2	-1	0
y	0	-1	0	3

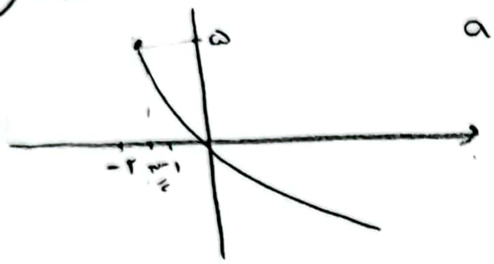


$R_f=[-1,+\infty)\cup(-1,0]=[-1,+\infty)$

۱۹

$y=\underbrace{a+1}_{\text{انتقال در راستای }y\text{ ها}}-\sqrt{2x+3}$

$2x+3\geq 0 \rightarrow 2x\geq -3 \rightarrow x\geq -\frac{3}{2} \rightarrow [\underbrace{-\tfrac{3}{2}}_{b},+\infty)$



$a+1=5 \rightarrow a=4$

$a\cdot b=\cancel{4}^{2}\times\frac{-3}{\cancel{2}}=-6$

۱۰

$f(x)=2\sqrt{x+1}+4\sqrt{1-x}$ $\quad D_f=D_g$

$f(x)+g(x)=3\sqrt{2+2\sqrt{1-x^2}}\overset{*}{=}3(\sqrt{1+x}+\sqrt{1-x})$

$\sqrt{2+2\sqrt{1-x^2}}=\sqrt{\underbrace{2}_{A}+\sqrt{\underbrace{4-4x^2}_{B}}}$ $\quad C=\sqrt{A^2-B}=\sqrt{4-(4-4x^2)}=\sqrt{4x^2}=2x$

$\sqrt{\frac{A+C}{2}}+\sqrt{\frac{A-C}{2}}=\sqrt{\frac{2+2x}{2}}+\sqrt{\frac{2-2x}{2}}=\sqrt{1+x}+\sqrt{1-x}$ (*)

$\Rightarrow g(x)=(f+g)(x)-f(x)=3\sqrt{1+x}+3\sqrt{1-x}-2\sqrt{x+1}-4\sqrt{1-x}=$

$\sqrt{1+x}-\sqrt{1-x}$

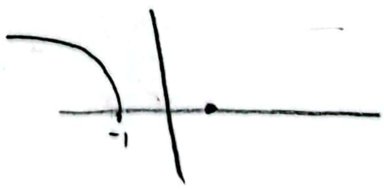
$\frac{f(x)}{6}=\frac{2}{6}\sqrt{x+1}+\frac{4}{6}\sqrt{1-x}=\frac{1}{3}\sqrt{x+1}+\frac{2}{3}\sqrt{1-x}$

$\frac{g(x)}{3}=\frac{1}{3}\sqrt{1+x}-\frac{1}{3}\sqrt{1-x}$

$y=\frac{f(x)}{6}-\frac{g(x)}{3}=(\frac{2}{3}+\frac{1}{3})\sqrt{1-x}=\sqrt{1-x}\geq 0$

$1-x\geq 0$

$x\leq 1$



$\sqrt{1-x}=\sqrt{-x+1}$

$R_{(\frac{f}{6}-\frac{g}{3})}=[0,+\infty)$