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$$f(x) = \sqrt{1-x^2} \text{ و } g(x) = \{(1,2), (2,0), (0,4), (-1,1)\}$$

$$2g - 3f = ? \rightarrow f(-1) = \sqrt{1-1} = 0 \rightarrow (-1,0) \text{ و } f(0) = \sqrt{1-0} = \sqrt{1} = 1 \rightarrow (0,1)$$

$$\text{و } f(2) = \sqrt{1-4} = \sqrt{-3} \text{ تن } x \text{ و } f(1) = \sqrt{1-1} = \sqrt{0} \rightarrow (1,0)$$

$$3f \rightarrow (-1,0), (0,3), (1,0) \text{ و } 2g \rightarrow (-1,2), (0,8), (1,4)$$

$$2g - 3f \rightarrow \{(1,4), (0,5), (-1,2)\}$$

۲

$$f(x) = 2x - 1 \xrightarrow{\text{دامنه}} [3, +\infty) \rightarrow f(3) = 5 \xrightarrow{\text{برد}} [5, +\infty)$$

$$g(x) = \frac{1}{3}x + 3 \xrightarrow{\text{دامنه}} (-\infty, 3] \rightarrow g(3) = 4 \xrightarrow{\text{برد}} (-\infty, 4]$$

$$\text{اتصال برد } \rightarrow (-\infty, 4] \cup [5, +\infty)$$

۳

$$y = -\frac{x^2}{2} + x + 3 \rightarrow \text{ولی: } -\frac{b}{2a} = \frac{-1}{-1} = 1 = x_{\max} \text{ و } y_{\max} = 3.5$$

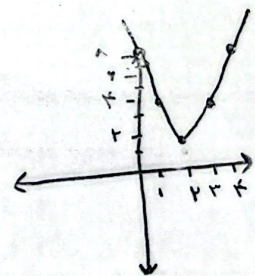
$$\rightarrow \text{می نویسیم } \rightarrow x = 3 \rightarrow -\frac{9}{2} + 3 + 3 = 1.5$$

$$\Rightarrow R_f \geq 1.5 \rightarrow D_f = (-1, 3) \rightarrow \sqrt{b-a} = \sqrt{3-(-1)} = \sqrt{4} = 2$$

۴

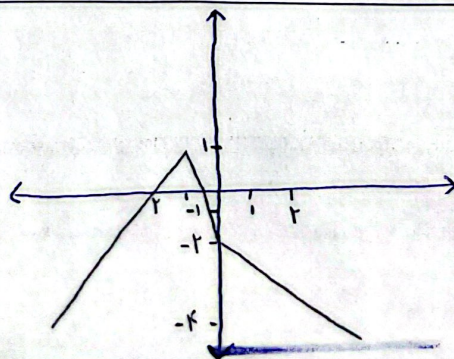
$$y = |x-1| + |x-3| + |2x-4| \xrightarrow{\text{بارها}} \begin{aligned} x < 1 &\rightarrow 1-4x \rightarrow y = 4 \\ 1 < x < 2 &\rightarrow 4-2x \rightarrow y = 4 \\ 2 < x < 3 &\rightarrow 2x-2 \rightarrow y = 4 \\ x > 3 &\rightarrow 4x-1 \rightarrow y = 4 \end{aligned}$$

$$\rightarrow \text{میزان حداقل} = 2$$



۵

$$y = |x| - 2|x+1|$$



$$\xrightarrow{\text{برد تابع}} (-\infty, 1]$$

$$y = \frac{x^2 + \omega x + m}{x+1} \rightarrow x^2 + \omega x + m = y(x+1) \rightarrow x^2 + (\omega - y)x + (m - y) = 0$$

$$\Delta = b^2 - 4ac = (\omega - y)^2 - 4(m - y) = y^2 - 4y + (\omega^2 - 4m) \geq 0$$

$$\min \rightarrow y = 1 \rightarrow 1 - 1 + \omega^2 - 4m = 14 - 4m$$

$$14 - 4m \geq 0 \rightarrow -4m \geq -14 \rightarrow m \leq 4 \rightarrow m = \{1, 2, 3, 4\}$$

$$F(x) = \begin{cases} x+1 & x \geq 1 \xrightarrow{x=1} F(1) = 1+1 = 2 \rightarrow [2, +\infty) \\ x^2 - 2x + 1 & 0 \leq x < 1 \rightarrow (x-1)^2 + 1 \rightarrow F(0) = 1, F(1) = 2 \rightarrow [1, 2) \\ |x| + 1 & x < 0 \rightarrow F(0) = 1 \rightarrow (1, +\infty) \end{cases}$$

$$\rightarrow [1, +\infty)$$

$$y = a + 1 - \sqrt{2x+3} \rightarrow 2x \geq -3 \rightarrow x \geq -\frac{3}{2} \xrightarrow{\text{دائري}} [-\frac{3}{2}, +\infty) \xrightarrow{\text{بسيط}} b = -\frac{3}{2}$$

$$a \rightarrow [b, +\infty)$$

$$a \rightarrow (-\infty, a]$$

$$\rightarrow a + 1 - \sqrt{2x+3} \rightarrow \sqrt{2(-\frac{3}{2})+3} \rightarrow a + 1 \rightarrow -\infty \rightarrow (-\infty, a+1]$$

$$a + 1 = a \rightarrow a = 1 \rightarrow ab = 1 \cdot -\frac{3}{2} = -\frac{3}{2}$$

$$F(x) = \begin{cases} x^2 + 4x + 3 & x < 0 \rightarrow (x+2)^2 - 1 \rightarrow F(-2) = -1, F(0) = 3 \rightarrow [-1, +\infty) \\ [2x] - 2x & x > 0 \rightarrow [t] - t = -\{t\} \rightarrow (-1, 0] \end{cases}$$

$$\rightarrow [-1, \infty)$$

$$F(x) = 2\sqrt{x+1} + 1\sqrt{1-x}$$

$$F(x) + g(x) = 2\sqrt{x+1} + 1\sqrt{1-x} \rightarrow g(x) = 2\sqrt{x+1} + 1\sqrt{1-x} - F(x)$$

$$D_f = D_g$$

$$\frac{F(x)}{4} - \frac{g(x)}{3} \rightarrow y = \frac{F(x)}{4} - \frac{1}{3}(2\sqrt{x+1} + 1\sqrt{1-x} - F(x)) \rightarrow \sqrt{x+1} + \frac{F(x)}{3}$$

$$\frac{F(x)}{4} - \sqrt{x+1} - \frac{1}{3}\sqrt{1-x} \rightarrow F(x) = 2\sqrt{x+1} + 1\sqrt{1-x} \rightarrow \frac{F(x)}{4} = \sqrt{x+1} + \frac{1}{4}\sqrt{1-x}$$

$$y = \sqrt{x+1} + \frac{1}{4}\sqrt{1-x} - \sqrt{x+1} - \frac{1}{4}\sqrt{1-x} \rightarrow y = \begin{cases} x = -1 \rightarrow y = 2 \\ x = 0 \rightarrow y = 1 \\ x = 1 \rightarrow y = \frac{1}{4} \end{cases} \rightarrow y = [2, 1, \frac{1}{4}]$$