

17/10

کتاب ریاضیات ۱
A

$$f(x) = \sqrt{1-x^2}$$

$$g = \{(-1,1), (0,0), (1,0)\}$$

$$g \circ f = \{(-1,1), (0,0), (1,0)\}$$

$$g = \{(-1,1), (0,0), (1,0)\}$$

$$f = \{(-1,0), (0,1), (1,0)\}$$

$$\Rightarrow \{(-1,0), (0,1), (1,0)\}$$

$$\Rightarrow -1 + 0 + 1 = 0$$

$$f(x) = x-1 \xrightarrow{D_f} [1, \infty) \Rightarrow x(x-1) = 0 \Rightarrow R_f = [0, +\infty)$$

$$g(x) = \frac{1}{x}x + 3 \xrightarrow{D_g} (-\infty, 3] \Rightarrow \frac{1}{x}(x) + 3 = \varepsilon \Rightarrow R_g = (-\infty, 3]$$

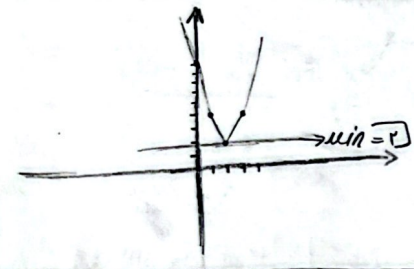
$$y = \frac{-x^2}{x} + x + 3 \rightarrow \max$$

$$\max \left| \frac{-b}{2a} = \frac{-1}{2(-1)} = 1 \right|$$

$$y > \frac{w}{x} \Rightarrow \left(\frac{-x^2}{x} + x + 3 \right) x > w \Rightarrow -x^2 + x^2 + 3x > w \Rightarrow 3x > w$$

$$\Rightarrow x^2 - x - \varepsilon < 0 \Rightarrow x = \frac{1 \pm \sqrt{1 - 4(-1)(-\varepsilon)}}{2(-1)} = \frac{1 \pm \sqrt{1 - 4\varepsilon}}{-2} = \frac{1 \pm \sqrt{1 - 4\varepsilon}}{-2}$$

$$\Rightarrow -1 < x < 3 \Rightarrow (a, b) = (-1, 3) \Rightarrow \sqrt{3+1} = 2$$

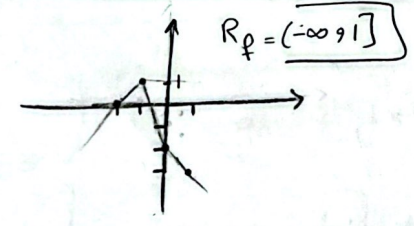


$$y = |x-1| + |x-2| + |x-3|$$

$x < 1$	$1 < x < 2$	$2 < x < 3$	$x > 3$
$-x+1 -x+2 -x+3 = -3x+6$	$x-1 -x+2 -x+3 = -x+4$	$x-1 x-2 -x+3 = 0$	$x-1 x-2 x-3 = x-6$
$[-\infty, 1]$	$[1, 2]$	$[2, 3]$	$[3, \infty)$

$$y = |x| - x|x+1|$$

$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
$-x - (-x)(-x-1) = -x - (-x^2 - x) = x^2$	$-x - (-x)(x-1) = -x - (-x^2 + x) = x^2 - 2x$	$-x - (-x)(x+1) = -x - (-x^2 - x) = x^2$	$-x - (-x)(x+1) = -x - (-x^2 - x) = x^2$
$[-\infty, -1]$	$[-1, 0]$	$[0, 1]$	$[1, \infty)$



$$y = \frac{x^2 + \omega x + m}{x+1} \Rightarrow x^2 + \omega x + m = (x+\varepsilon)(x+1) + (m-\varepsilon)$$

$$\Rightarrow y(x+1) = (x+\varepsilon)(x+1) + (m-\varepsilon)$$

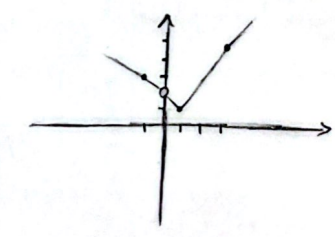
$$\Rightarrow yx + y = x^2 + \omega x + m \Rightarrow x^2 + (\omega - y)x + (m - y) = 0$$

$$\Delta = (\omega - y)^2 - 4(m - y) \Rightarrow \Delta = y^2 - 2y\omega + \omega^2 - 4m + 4y$$

$$\Rightarrow \Delta < 0 \Rightarrow (-y)^2 - 4(\omega - \varepsilon)m < 0 \Rightarrow m < \varepsilon \Rightarrow 1 < \varepsilon$$

$$f(x) = \begin{cases} x+r & ; x > r \\ x^2 - x + r & ; 0 \leq x \leq r \\ |x| + r & ; x < 0 \end{cases}$$

$x < 0$	$0 \leq x \leq r$	$x > r$
$-x+r$	$x^2 - x + r$	$x+r$
$[-\infty, 0]$	$[0, r]$	$[r, \infty)$

$$R_f = [1, +\infty)$$


$$f(x) = \begin{cases} x^2 + \varepsilon x + \varepsilon & ; x \leq 0 \rightarrow \min \left| \frac{-b}{2a} = -\frac{\varepsilon}{2} = -\frac{\varepsilon}{2} \Rightarrow [-1, +\infty) \end{cases}$$

$$1 \leq 2x < 1 + 1 \Rightarrow \begin{cases} 2x = 1 \Rightarrow 1 - 1 \leq 0 \\ 1 - (1 + 1) = -1 \Rightarrow (-1, 0] \end{cases} \Rightarrow R_f = [-1, +\infty)$$

$$y = a + 1 - \sqrt{2x + \varepsilon} \Rightarrow 2x + \varepsilon \geq 0 \Rightarrow x \geq -\frac{\varepsilon}{2} \Rightarrow [-\frac{\varepsilon}{2}, +\infty) \Rightarrow b = -\frac{\varepsilon}{2}$$

$$D_f = [b, +\infty) \quad a + 1 - \sqrt{2x - \frac{\varepsilon}{2} + \varepsilon} \Rightarrow a + 1 = a \Rightarrow a = \varepsilon$$

$$R_f = (-\infty, a] \quad \frac{\varepsilon}{2} x = \frac{\varepsilon}{2} = -\varepsilon$$

$$ab = ?$$

$$f(x) = \varepsilon \sqrt{x+1} + \varepsilon \sqrt{1-x} \Rightarrow \begin{cases} x+1 \geq 0 \Rightarrow x \geq -1 \\ 1-x \geq 0 \Rightarrow 1 \geq x \end{cases} \Rightarrow [-1, 1] = D_f = D_g$$

$$\frac{f(x)}{\varepsilon} - \frac{g(x)}{\varepsilon} = \frac{\varepsilon \sqrt{x+1} + \varepsilon \sqrt{1-x} + \varepsilon \sqrt{1+x} - \varepsilon \sqrt{1-x}}{\varepsilon} = \frac{\varepsilon \sqrt{1+x} + \varepsilon \sqrt{1-x}}{\varepsilon}$$

$$= \frac{\varepsilon (\sqrt{1-x} + \varepsilon \sqrt{1+x})}{\varepsilon} = \frac{\varepsilon \sqrt{1+x} + \varepsilon \sqrt{1-x}}{\varepsilon} \Rightarrow D_f = [-1, 1]$$

$$\text{if } x = -1 \Rightarrow \frac{\varepsilon \sqrt{-1+1} + \varepsilon \sqrt{1+1}}{\varepsilon} = \frac{\varepsilon \sqrt{2}}{\varepsilon}$$

$$\text{if } x = 1 \Rightarrow \frac{\varepsilon \sqrt{1+1} + \varepsilon \sqrt{1-1}}{\varepsilon} = \frac{\varepsilon \sqrt{2}}{\varepsilon}$$

$$\Rightarrow R_f = \left[\frac{\varepsilon \sqrt{2}}{\varepsilon}, \frac{\varepsilon \sqrt{2}}{\varepsilon} \right]$$

$$f(x) + g(x) = \varepsilon \sqrt{1+x} + \varepsilon \sqrt{1-x} \xrightarrow{\text{...}} \varepsilon (\sqrt{1-x} + \sqrt{1+x})$$

$$\rightarrow g(x) = \sqrt{1+x} - \sqrt{1-x}$$

$$\frac{f(x)}{\varepsilon} - \frac{g(x)}{\varepsilon} = \sqrt{1-x}$$

