

1 $f(x) = \sqrt{1-x^2} \Rightarrow D_f \Rightarrow 1-x^2 \geq 0 \quad x^2 \leq 1 \quad -1 \leq x \leq 1 \quad D_f \cap D_g = \{ -1, 0, 1 \}$

$f_g = \{ (-1, 2)(0, 1)(1, 0)(1, 4) \} \quad f_f = \{ (-1, 0)(0, 3)(1, 0) \}$

$f_g - f_f = \{ (-1, 2)(0, 1)(1, 4) \}$

2 $f(x) = 2x - 1 \rightarrow D_f = [2, +\infty) \quad f(2) = 3 \quad R_f = [3, +\infty)$

$g(x) = \frac{1}{x} x + 3 \rightarrow D_g = (-\infty, 3] \quad f(3) = 5 \quad R_g = (-\infty, 5]$

$R_f \cup R_g = (-\infty, 5] \cup [3, +\infty)$

3 $f(x) > \frac{x}{x} \quad \frac{-x^2}{x} + x + 3 > \frac{x}{x} \xrightarrow{\text{ساده شود}} \frac{-x^2}{x} + x + 3 = \frac{x}{x} \xrightarrow{\times x} -x^2 + 2x + 9 > 1$

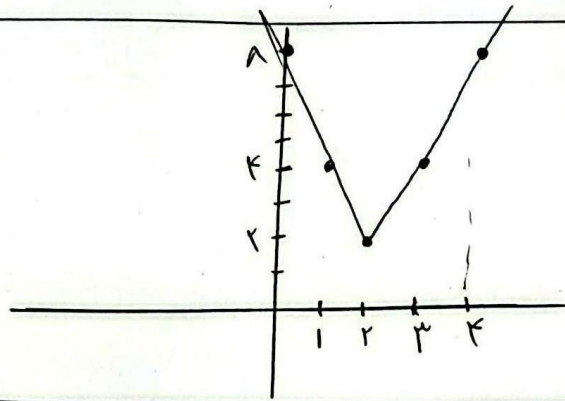
$-x^2 + 2x + 9 > 1 \Rightarrow -x^2 + 2x + 8 > 0 \quad x^2 - 2x - 8 < 0$

$(x-4)(x+2) < 0 \quad (a, b) = (-2, 4) \leftarrow \text{برای } x \text{ در این بازه جواب می‌دهد}$

$a = -2 \quad b = 4 \quad b - a = 4 - (-2) = 6 \quad \sqrt{6} = \boxed{2.45}$

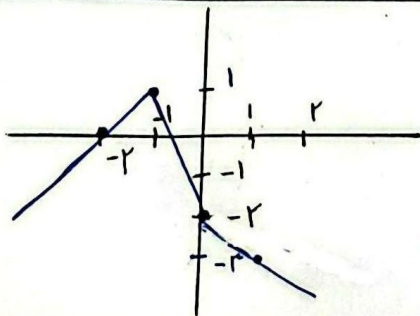
4 $y = |x-1| + |x-2| + |2x-4|$

نقطه مینیمم $x=2$



5 $y = |x| - 2|x+1|$

$R = (-\infty, 1]$



6 $y = \frac{x^2 + ax + m}{x+1} \Rightarrow x^2 + ax + m = y(x+1) \Rightarrow x^2 + (a-y)x + (m-y) = 0$

$\Delta = b^2 - 4ac = (a-y)^2 - 4(m-y) = y^2 - 4y + (a^2 - 4m) \geq 0$

$y^2 - 4y + (a^2 - 4m) \geq 0$

$\min \Rightarrow y = 2 \Rightarrow 4 - 8 + a^2 - 4m = 0 \Rightarrow a^2 - 4m = 4$

$4 - 4m \geq 0 \quad 4 \geq 4m \Rightarrow \text{if } m = 1 \quad R = [1, 2] \quad x = -1$

در نهایت $m < 4$
اعداد صحیح که می‌تواند از ۴ باشد:
 $m = 1, 2, 3$
(عدد ۳)

$$f(x) = \begin{cases} x+2 & x \geq 2 \rightarrow f(2) = 4 \rightarrow [4, +\infty) \\ x^2 - 2x + 2 & 0 \leq x < 2 \rightarrow f(0) = 2 \quad f(2) = 4 \text{ (نقطه) } = (2, 4) \rightarrow [2, 4) \\ |x| + 2 & x < 0 \rightarrow f(0) = 2 \rightarrow (2, +\infty) \end{cases}$$

$$f(x) \geq 2 \Rightarrow R_f = [2, +\infty)$$

$$f(x) = \begin{cases} x^2 + 2x + 2 : x \leq 0 \Rightarrow \text{نقطه بحرانی} = \frac{-b}{2a} = \frac{-2}{2} = -1 \quad y_s = -1 \\ [2x] - 2x : x > 0 \Rightarrow R = (-1, 0] \end{cases} \quad [-1, +\infty)$$

$$\rightarrow \therefore [-1, +\infty)$$

$$y = a + 1 - \sqrt{2x + 2} \quad D \Rightarrow 2x + 2 \geq 0 \quad x \geq -1 \quad b = -1$$

نقطه بحرانی $x = -1$ که داخل دامنه است و مقدار صفر شود

$$y = a + 1 \rightarrow \text{نقطه بحرانی}$$

$$a + 1 = 0 \quad a = -1 \quad a \times b = -1 \times -1 = 1$$

$$g(x) = 2\sqrt{2+2\sqrt{1-x^2}} - f(x) \quad f(x) = 2\sqrt{x+1} + 2\sqrt{1-x}$$

$$y = \frac{f(x)}{4} - \frac{g(x)}{2} = \frac{f(x)}{4} - \frac{2\sqrt{2+2\sqrt{1-x^2}} - f(x)}{2} =$$

$$\frac{2f(x) - 4\sqrt{2+2\sqrt{1-x^2}}}{4} = \frac{f(x)}{2} - \sqrt{2+2\sqrt{1-x^2}} =$$

$$\frac{2(\sqrt{x+1} + 2\sqrt{1-x})}{2} - \sqrt{2+2\sqrt{1-x^2}} = \sqrt{x+1} + 2\sqrt{1-x} - \sqrt{2+2\sqrt{1-x^2}}$$

$$\rightarrow \therefore \quad \begin{aligned} & \text{نقطه بحرانی: } x+1 \geq 0 \quad x \geq -1 \\ & 1-x \geq 0 \quad x \leq 1 \end{aligned} \Rightarrow b = [-1, 1]$$

$$y = \begin{cases} x = -1 \Rightarrow y = \sqrt{2} - 2 \\ x = 1 \Rightarrow \sqrt{2} - 2 \end{cases} \Rightarrow R = [\sqrt{2} - 2, \sqrt{2} - 2]$$