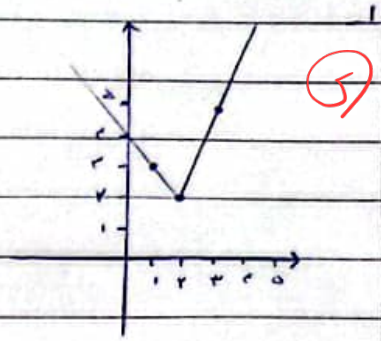
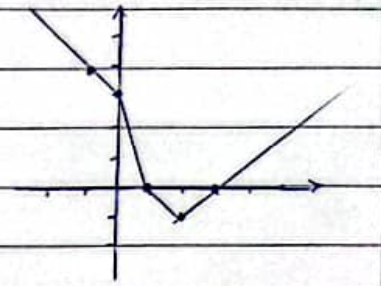
الف) $y = x + |2x - 4|$ ⟶ ریشه قدرمطلق = ۲ ⟶ $\begin{vmatrix}1\\3\end{vmatrix}\ \begin{vmatrix}2\\2\end{vmatrix}\ \begin{vmatrix}3\\5\end{vmatrix}$

۱



$R_f = [2, +\infty)$

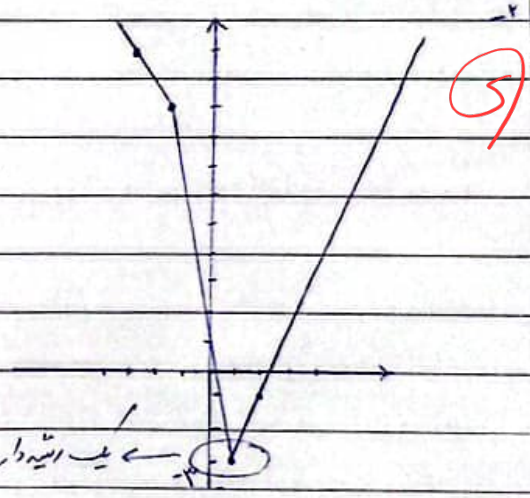
ب) $y = |x-2| + |x-1| - |x|$ ⟶ $\begin{vmatrix}-1\\4\end{vmatrix}\ \begin{vmatrix}0\\3\end{vmatrix}\ \begin{vmatrix}1\\0\end{vmatrix}\ \begin{vmatrix}2\\-1\end{vmatrix}\ \begin{vmatrix}3\\0\end{vmatrix}$



$R_f = [-1, +\infty)$

$|3x-3| - |x+2| = K$ ⟶ $\begin{vmatrix}-3\\11\end{vmatrix}\ \begin{vmatrix}-2\\9\end{vmatrix}\ \begin{vmatrix}1\\-3\end{vmatrix}\ \begin{vmatrix}2\\-1\end{vmatrix}$

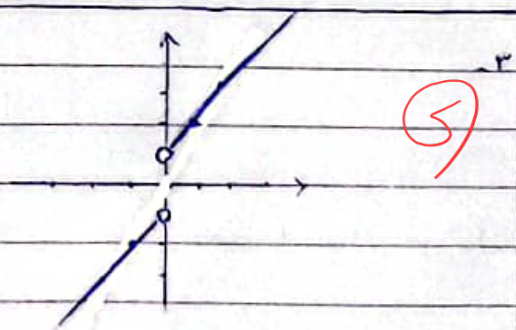
۲



$\Rightarrow K = -3$

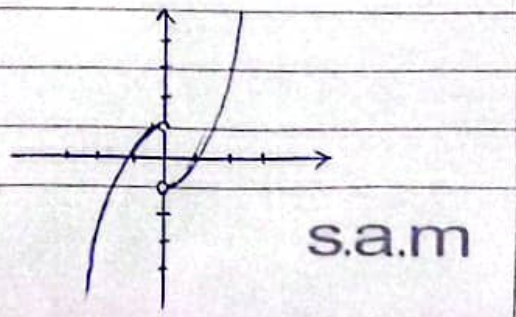
الف) $y = x + \frac{x}{|x|}$ ⟶ $\begin{cases} x+1 & ; x > 0 \\ x-1 & ; x < 0 \end{cases}$

۳



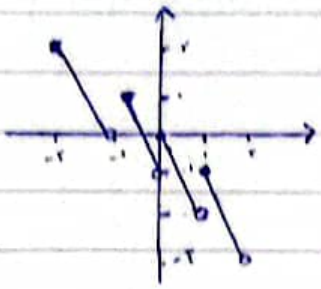
$D = \mathbb{R} - \{0\}$ $R_f = \mathbb{R} - [-1, 1]$

ب) $x|x| - \frac{x}{|x|}$ ⟶ $\begin{cases} x^2 - 1 & ; x > 0 \\ -x^2 + 1 & ; x < 0 \end{cases}$

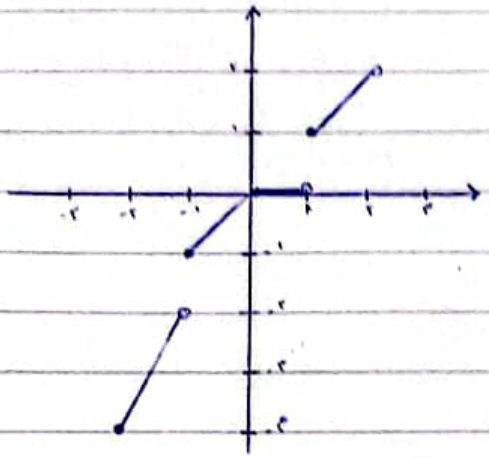


$D = \mathbb{R} - \{0\}$ $R_f = \mathbb{R}$

۶- الف) $g=[x]-2x$



ب) $g=[x]|x|$



۵

الف) $g=3x-[3x] \quad 0 \leq x-[x]<1 \rightarrow R_f=[0,1)$

ب) $g=4x-4[x] \rightarrow y=4(x-[x]) \rightarrow R_f=[0,4)$

ج) $g=x-5\left[\frac{x}{5}\right] \rightarrow y=5\left(\frac{x}{5}-\left[\frac{x}{5}\right]\right) \rightarrow R_f=[0,5)$

د) $g=[2x]-2x \rightarrow g=-(2x-[2x]) \rightarrow R_f=(-1,0]$

۷- الف) $g=3\sin x+2\cos x \rightarrow -\sqrt{9+4} \leq 3\sin x+2\cos x \leq +\sqrt{9+4} \rightarrow R_f=[-\sqrt{13},\sqrt{13}]$

ب) $y=4\sin x-3\cos x \rightarrow -\sqrt{16+9} \leq 4\sin x-3\cos x \leq +\sqrt{16+9} \rightarrow R_f=[-5,5]$

ج) $g=[3\sin x+5\cos x] \rightarrow -\sqrt{34} \leq 3\sin x+5\cos x \leq +\sqrt{34} \rightarrow R_f=\{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

د) $g=\sqrt{\cos x-2\sin x} \rightarrow -\sqrt{5} \leq \cos x-2\sin x \leq \sqrt{5} \rightarrow R_f=[0,\sqrt[4]{5}]$

۸- الف) $\sin^2 x+3\sin x+2=y \quad \sin x=-1 \rightarrow y=0 \min$

$\sin x=1 \rightarrow y=6 \max \quad R_f=[0,6]$

$\sin x=\frac{-b}{2a}=-\frac{3}{2} \rightarrow \times$

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ب) $y = 2\sin^2 x + 3\sin x + 2$ $\quad \sin x = -1 \rightarrow y = 1$

$\sin x = 1 \rightarrow y = 7$ max $\quad R_f = [\frac{7}{8}, 7]$

$\sin x = \frac{-b}{2a} = \frac{-3}{4} \rightarrow y = \frac{7}{8}$ min

ج) $y = \sin^2 x + 4\cos^2 x = \sin^2 x + 4 - 4\sin^2 x = 4 - 3\sin^2 x$ ـ ۸

$0 \leq \sin^2 x \leq 1 \rightarrow 0 \geq -3\sin^2 x \geq -3 \rightarrow 4 \geq 4 - 3\sin^2 x \geq 1 \rightarrow R_f = [1, 4]$

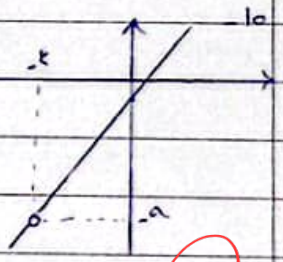
د) $y = \frac{2\cot x}{1 + \cot^2 x} \Rightarrow y = \frac{2\cos x}{\sin x} \times \sin^2 x = 2\sin x \cos x = \sin 2x \rightarrow R_f = [-1, 1]$

ه) $y = \sin^4 x + \cos^4 x \rightarrow \frac{1}{2} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow R_f = [\frac{1}{2}, 1]$ ـ ۹

و) $y = [\sin^8 x + \cos^8 x] \rightarrow \frac{1}{8} \leq \sin^8 x + \cos^8 x \leq 1 \rightarrow R_f = \{0, 1\}$

ز) $y = \frac{2x^2 + 7x - 4}{x + 4} = \frac{(2x - 1)(x + 4)}{(x + 4)} = 2x - 1$

$x \neq -4 \Rightarrow y \neq 2(-4) - 1 \rightarrow y \neq -9 \Rightarrow R_f = \mathbb{R} - \{-9\}$

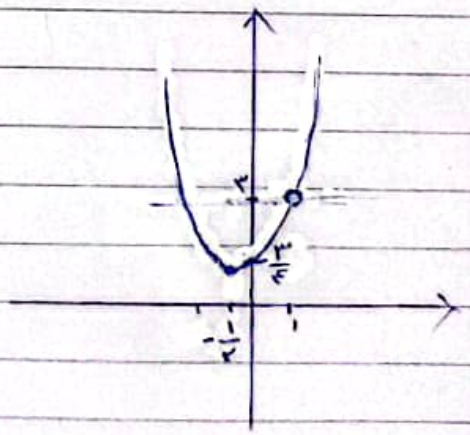


ح) $y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + x + 1)}{(x-1)} = x^2 + x + 1$

$x \neq 1 \rightarrow y \neq 1 + 1 + 1 \rightarrow y \neq 3$

$y = x^2 + x + 1 \rightarrow \begin{cases} x_{min} = -\frac{b}{2a} = -\frac{1}{2} \\ y_{min} = \frac{3}{4} \end{cases}$

$\Rightarrow R_f = [\frac{3}{4}, +\infty)$



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