نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ...۷... کلاس .....A.....

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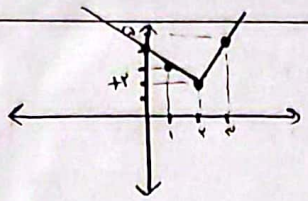
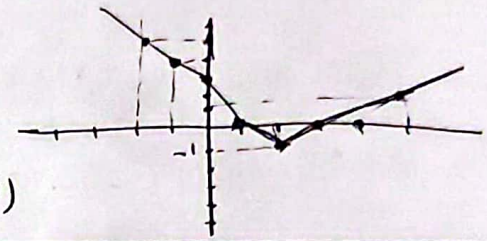
الف)
$$\begin{cases} x \geq 2 & x^2 - 2 \\ x < 2 & -x + 4 \end{cases}$$



$Rf = [2, +\infty)$

ب)

$-\infty$		0		1		2		$+\infty$
$-x+2$	$-x^2+2$		$-x^2+2$		$-x+1$		$x-2$	



$Rf = [-1, +\infty)$

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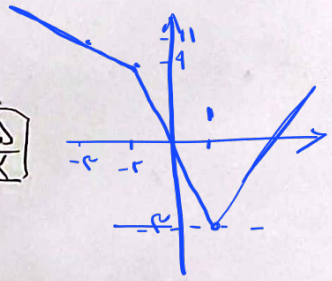
$|x^2-2| - |x+2| = k$ به ازای $x = 1, -2$

$$\begin{cases} x > 1 & x^2 - 2 \to \frac{5}{2} \\ -2 \leq x \leq 1 & -x + 1 \to \frac{1}{2} \\ x < -2 & -x^2 + 2 \to \frac{5}{2} \end{cases} \longrightarrow \boxed{k = \frac{5}{2}}$$



$k = -3$

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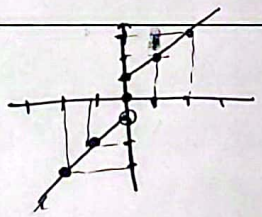
الف)
$$\begin{cases} x \geq 0 & x + 1 \\ x < 0 & x - 1 \end{cases}$$



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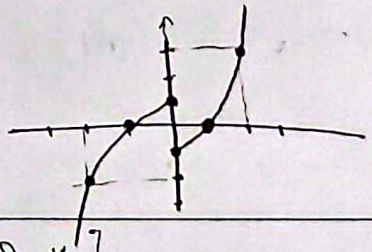
ب)
$$\begin{cases} x \geq 0 & x^2 - 1 \\ x < 0 & -x^2 + 1 \end{cases}$$



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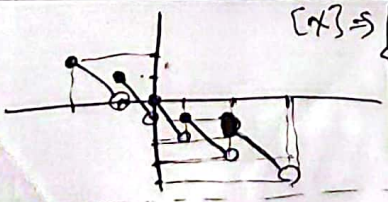
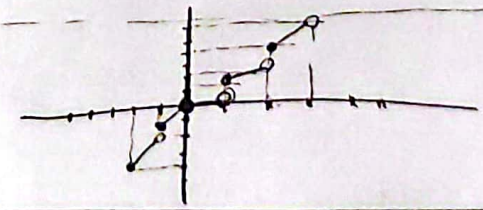
الف) ① $-2 \leq x < -1$ ② $-1 \leq x < 0$ ③ $0 \leq x < 1$ ④ $1 \leq x < 2$ ⑤ $2 \leq x < 3$

$[x] \Rightarrow$ ① $= -2$ ② $= -1$ ③ $= 0$ ④ $= 1$ ⑤ $= 2$



ب)
$-2 \leq x < -1 \Rightarrow [x] = x+2$
$-1 \leq x < 0 \Rightarrow [x] = x$
$0 \leq x < 1 \Rightarrow [x] = 0$
$1 \leq x < 2 \Rightarrow [x] = x$
$2 \leq x < 3 \Rightarrow [x] = x+2$



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الف) $y = x - [x] \Rightarrow Rf = [0, 1)$

نمودار $0 \leq x - [x] < 1$

ب) $y = x - [x] \Rightarrow y = (x - [x]) \Rightarrow Rf = [0, 4)$ ($[0,1)$)

ج) $y = x - 5\left[\frac{x}{5}\right] \Rightarrow 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right) \Rightarrow Rf = [0, 5)$ ($[0,1)$)

د) $y = [2x] - 2x \Rightarrow Rf = (-1, 0]$

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أ) $-\sqrt{9+4} \le 2\sin x + 3\cos x \le \sqrt{9+4} \Rightarrow -\sqrt{13} \le 2\sin x + 3\cos x \le \sqrt{13} = R_f$

مرفوض $\in -\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$

ب) $-\sqrt{25} \le 4\sin x - 3\cos x \le \sqrt{25} \Rightarrow -5 \le 4\sin x - 3\cos x \le 5 = R_f$

ج) $-\sqrt{25} \le 4\sin x + 3\cos x \le \sqrt{25} \Rightarrow -5 \le [4\sin x + 3\cos x] \le 5 \Rightarrow R_f = \{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

د) $-\sqrt{5} \le \cos x - 2\sin x \le \sqrt{5} \Rightarrow 0 \le \sqrt{\cos x - 2\sin x} \le \sqrt{5} = R_f$

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الف) $\sin^2 x + 2\sin x + 2 \Rightarrow [\sin x]$ max=1 ⟶ $1+2+2=5$; min=-1 ⟶ $1-2+2=0$; $-\frac{b}{2a} = -\frac{2}{2} = -1$; $\frac{9}{4} - \frac{6}{4} + 2 = 2$

$R_f = [0,5]$

ب) $2\sin^2 x + 2\sin x + 2 \Rightarrow [\sin x]$ max=1 ⟶ $2+2+2=6$; min=-1 ⟶ $2-2+2=2$; $-\frac{b}{2a} = \frac{1}{4}$; $\frac{12}{8} - \frac{6}{4} + 2 = \frac{12}{8} = \frac{3}{2}$

$R_f = [\frac{3}{2}, 6]$

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الف) $y = \sin^2 x + 4\cos^2 x \Rightarrow \sin^2 x + \cos^2 x + 3\cos^2 x \Rightarrow 1 + 3\cos^2 x \Rightarrow$

$0 \le \cos^2 x \le 1 \Rightarrow 0 \le 3\cos^2 x \le 3 \Rightarrow 1 \le 3\cos^2 x + 1 \le 4 \Rightarrow R_f = [1,4]$

ب) $y = \frac{2\cot x}{1+\cot^2 x} = \frac{2 \times \frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = \frac{2\cos x \times \sin^2 x}{\sin x} \Rightarrow 2\cos x \sin x = \sin 2x$

$-1 \le \sin 2x \le 1 \Rightarrow R_f = [-1,1]$

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الف) $k=2 \Rightarrow 2^{1-2} \le \sin^2 x + \cos^2 x \le 2 \Rightarrow \frac{1}{2} \le \sin^2 x + \cos^2 x \le 2 \Rightarrow R_f = [\frac{1}{2}, 1]$

ب) $k=4 \Rightarrow 2^{1-4} \le \sin^4 x + \cos^4 x \le 2 \Rightarrow \frac{1}{8} \le \sin^4 x + \cos^4 x \le 2 \Rightarrow 0 \le [\sin^4 x + \cos^4 x] \le 1 \Rightarrow R_f = \{0,1\}$

مرفوض $\in 2^{1-k} \le \sin^{2k} x + \cos^{2k} x \le 2$

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الف) $yx + 2y = 2x^2 + 3x - 2 \Rightarrow \Delta \ge 0 \Rightarrow 0x^2 + (3-y)x - 2 - 2y \Rightarrow$

$\Delta \Rightarrow (3-y)^2 - 4(0)(-2-2y) \ge 0 \Rightarrow y^2 + 18y + 81 \ge 0 \Rightarrow (y+9)^2 \ge 0$

$R_f = \mathbb{R}$

ب) $y = \frac{x^3-1}{x-1} = \frac{(x-1)(x^2+1+x)}{(x-1)} \Rightarrow x^2 + x + 1 = y$; $x \ne 1$, $y \ne 3$

$\frac{-b}{2a} \Rightarrow \frac{-1}{2} \rightarrow R_f = [\frac{3}{4}, +\infty) - \{3\}$