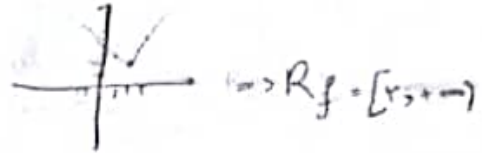


الف) $y = x + |x - 2|$

$$\left. \begin{aligned} x - 2 \geq 0 &\Rightarrow x \geq 2 \rightarrow x + x - 2 = 2x - 2 \\ x - 2 < 0 &\Rightarrow x < 2 \rightarrow x - x + 2 = -x + 2 \\ x - 2 = 0 &\Rightarrow x = 2 \rightarrow x = 2 \end{aligned} \right\}$$



5-1

ب) $y = |x - 1| + |x + 1| - |x|$

$$\left. \begin{aligned} x > 0 &\rightarrow y = -x + 2 \\ 1 < x < 0 &\rightarrow y = -2x + 2 \\ 0 < x < 1 &\rightarrow y = -x + 1 \\ x < 0 &\rightarrow y = x - 2 \end{aligned} \right\} \Rightarrow [-1, +\infty)$$

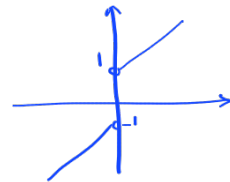
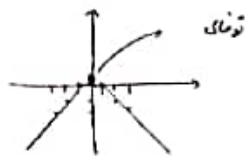
$|x - 1| - |x + 1| = k \Rightarrow x|x - 1| - |x + 1| = k$

$$\left. \begin{aligned} x < -1 &\rightarrow y = -2x + 0 \\ -1 < x < 1 &\rightarrow y = -2x + 1 \\ x > 1 &\rightarrow y = 2x - 0 \end{aligned} \right\} \rightarrow \text{برای اینکه دایره با خط مماس باشد باید } k = 0 \text{ باشد.} \rightarrow \boxed{k = 0, -3}$$

5-2

الف) $y = x + \frac{x}{|x|}$

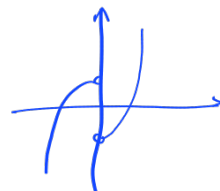
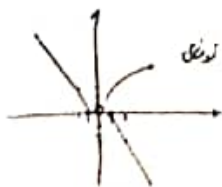
$$\left. \begin{aligned} x > 0 &\rightarrow y = 1 + x \\ x < 0 &\rightarrow y = -1 + x \end{aligned} \right\}$$



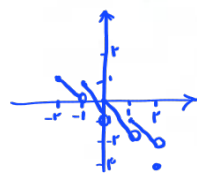
5-3

ب) $y = x|x| - \frac{x}{|x|}$

$$\left. \begin{aligned} x > 0 &\rightarrow y = x^2 - 1 \\ x < 0 &\rightarrow y = -x^2 - 1 \end{aligned} \right\}$$



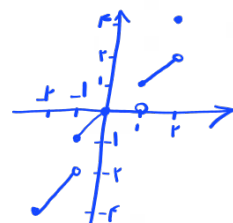
$$\begin{aligned} x < -1 &\rightarrow y = -2x \\ -1 < x < 1 &\rightarrow y = 2x - 1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x = -1 &\rightarrow y = 2 \\ x = 1 &\rightarrow y = 0 \\ x = -1 &\rightarrow y = 1 \\ x = 0 &\rightarrow y = 0 \\ x = 1 &\rightarrow y = -1 \end{aligned}$$



1/2

$$\begin{aligned} -1 < x < -1 &\rightarrow y = 2x \\ -1 < x < 0 &\rightarrow y = 2 \\ 0 < x < 1 &\rightarrow y = 0 \\ 1 < x < 2 &\rightarrow y = 1 \end{aligned}$$

$$\Rightarrow \begin{aligned} x = -1 &y = -2 \\ x = 0 &y = -1 \\ x = 1 &y = 1 \\ x = 2 &y = 2 \end{aligned}$$



$$\text{مثال } y = \lfloor x \rfloor - \lceil x \rceil \rightarrow \text{الدالة غير متصلة في كل نقطة} \Rightarrow R_f = [0, 1)$$

$$\text{مثال } f(x) = \lceil x \rceil \rightarrow \lceil x - \lfloor x \rfloor \rceil \Rightarrow R_f = [0, 1)$$

$$\text{مثال } x = a \lfloor \frac{x}{a} \rfloor \rightarrow a \left(\frac{x}{a} - \lfloor \frac{x}{a} \rfloor \right) = a \times \left\{ \frac{x}{a} \right\} \Rightarrow R_f = [0, a)$$

$$\text{مثال } \lceil x \rceil - x \rightarrow \lceil \lfloor x \rfloor - x \rceil = (-1, 0]$$

$$\text{مثال } y = r \sin x + r \cos x \rightarrow -\sqrt{r^2 + r^2} = -\sqrt{14} \rightarrow R_f = [-\sqrt{14}, \sqrt{14}]$$

$$\text{مثال } y = r \sin x - r \cos x \rightarrow -\sqrt{r^2 + r^2} = -\sqrt{10} \rightarrow R_f = [-\sqrt{10}, \sqrt{10}]$$

$$\text{مثال } y = [r \sin x + a \cos x] \rightarrow -\sqrt{r^2 + a^2} = -\sqrt{19} \rightarrow R_f = [-\sqrt{19}, \sqrt{19}]$$

$$\text{مثال } y = \lfloor \cos 2 - r \sin x \rfloor \quad \cos x - r \sin x \rightarrow -\sqrt{1+1} = -\sqrt{2} \rightarrow R_f = [-\sqrt{2}, \sqrt{2}] \rightarrow R_f = [-\sqrt{2}, \sqrt{2}]$$

$y = \sin^2 x + r \sin x + 1 \Rightarrow (\sin x + 1)^2 + 1$
 $\frac{(\sin x + 1)^2 + 1}{r} + 1 = 1 \Rightarrow R_f = \left[\frac{1}{r}, \frac{1}{r} \right]$
 $\frac{(\sin x + 1)^2 + 1}{r} + 1 = 1 \Rightarrow \frac{1}{r} = 0$

1, 1/0

$y = r \sin^2 x + r \sin x + 1 \Rightarrow r \frac{1}{r} + r \frac{1}{r} + 1 \Rightarrow \frac{1}{r} = \frac{1}{r} \Rightarrow R_f = \left[\frac{1}{r}, \frac{1}{r} \right]$
 $R_f = \left[\frac{1}{r}, \frac{1}{r} \right]$

$y = \sin^2 x + r \cos^2 x = r \cos^2 x + 1 \Rightarrow x = 0 \rightarrow 1$
 $x = \pi \rightarrow r$
 $\Rightarrow R_f = [1, r]$

5

$y = \frac{r \cot x}{1 + \cot^2 x} = \frac{r \cot x}{\frac{1}{\sin^2 x}} = r \cot x \sin^2 x = r \cos x \sin x = \sin 2x \Rightarrow R_f = [-1, 1]$

$y = \sin^2 x + \cos^2 x \rightarrow r^{-1} = r^{-1} = \frac{1}{r} \Rightarrow R_f = \left[\frac{1}{r}, 1 \right]$

5

$y = [\sin^2 x + \cos^2 x] \rightarrow r^{-1} = r^{-1} = \frac{1}{r} \Rightarrow [\sin^2 x + \cos^2 x] \Rightarrow R_f = \{0, 1\}$

$y = \frac{rx^2 + vx - r}{x + r} \Rightarrow \text{division} \rightarrow y = rx - 1 \rightarrow r(-r) - 1 = -r^2 - 1 \Rightarrow R_f = \mathbb{R} - \{-r^2 - 1\}$

5

$y = \frac{x^2 - 1}{x - 1} \Rightarrow y = \frac{(x-1)(x+1)}{x-1} \Rightarrow y = x + 1 \Rightarrow \frac{1}{r} = \frac{1}{r}$

$\mathbb{R}_f = \left[\frac{1}{r}, +\infty \right) \quad C = y = \frac{1}{r} - \frac{1}{r} + 1 = \frac{r}{r}$