درسا صادقی — ۱۵,۵

سوال ۱) مرتب نوشته و عکس گرفته شود

الف) $x+|2x-4|$

$2x-4\geq 0$

$2x\geq 4 \rightarrow x\geq 2 \rightarrow R=[2,+\infty)$

$2x-4<0$

$2x<4$

$x<2$

$R=(2,+\infty)$

R نهایی $=(2,+\infty)$

۳

ب) $|x-2|+|x-1|-|x|$

$x<0$

$-x+2+-x+1-(-x)=-x+2-x+1+x=-x+3$

$0\leq x<1$

$-x+2+(-x+1)-x=-3x+3$

$1\leq x<2$

$-x+2+x-1-x=-x+1$

$x\geq 2 \longrightarrow (x-2)+(x-1)-x=x-3$

$R=[-1,+\infty)$

سوال ۲) ۱,۷۵

$k=|3x-3|-|x+2|$

$3x-3=0 \longrightarrow x=1 \qquad x+2 \longrightarrow x=-2$

$x<-2 \longrightarrow -3x+3+x-2=-2x+5$

$-2\leq x<1 \longrightarrow -3x+3+x+2=-4x+1$

$x\geq 1 \longrightarrow k=3x-3-x-2=2x-5$

$R=[-3,+\infty)$

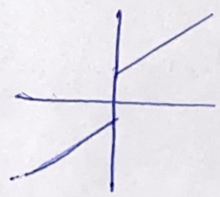
$k=-3$

سوال ۳)

الف) $x+\frac{x}{|x|} \rightarrow x\neq 0$

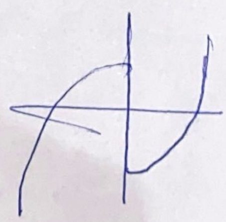
$|x|=\begin{cases} x & x\geq 0 \\ -x & x<0 \end{cases}$

۵



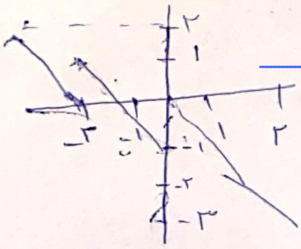
ب) $x|x|-\frac{x}{|x|}$

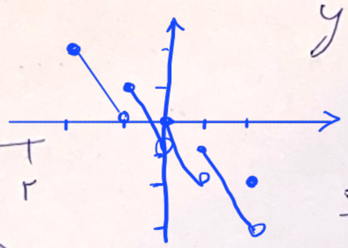
$x\neq 0$



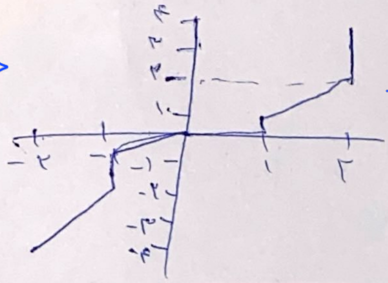
سوال ۴)

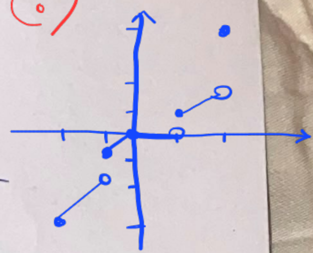
$y = [x] - 2x$





$y = [x]|x|$





۵

سوال ۵)

۶

الف) $y = 3x - [3x]$

$0 \le y < 1 \qquad R = [0,1)$

ب) $y = 4x - 4[x] \longrightarrow 4(x - [x])$

$0 \le x - [x] < 1 \quad \times 4$

$0 \le y < 4 \longrightarrow R = [0,4)$

ج) $y = x - 5\left[\frac{x}{5}\right] \quad \times 5 \qquad R = [0,5)$

د) $y = [2x] - 2x \qquad 0 \le 2x\ldots < 1 \qquad -1 < y \le 0 \qquad R = (-1,0]$

---

۶)

۶

الف) $y = 3\sin x + 2\cos x \longrightarrow -\sqrt{13} \le y \le \sqrt{13}$

$R = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin x - 3\cos x \longrightarrow -\sqrt{25} \le y \le \sqrt{25} \longrightarrow R = [-5,5]$

ج) $y = [2\sin x + 5\cos x] \longrightarrow -\sqrt{29} \le y \le \sqrt{29} \longrightarrow R = \cancel{[\ldots]}$

$\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

د) $y = \sqrt{\cos x - 2\sin x} \ge 0$

$-\sqrt{5} \le \ldots \le \sqrt{5} \longrightarrow 0 \le y \le \sqrt{\sqrt{5}} \longrightarrow R = [0, \sqrt{\sqrt{5}}]$

(درسا صادقی)

۷)

الف) $\sin^2 x + 3\sin x + 2$

$y^2 + 3y + 2 \qquad \sin x = [-1,1]$

$2y + 3 = 0 \to y = -\frac{3}{2} \notin [-1,1]$

$y = -1 \to 1 - 3 + 2 = 0 \qquad R = [0,6]$

$y = 1 \to 1 + 3 + 2 = 6$

ب) $y = 2\sin^2(x) + 3\sin x + 2$

$2t^2 + 3t + 2 \qquad t \in [-1,1]$

$t = -1 = y = 2 - 3 + 2 = 1$

$t = 1 = y = 2 + 3 + 2 = 7 \qquad R = [\frac{7}{8}, 7]$

$4t + 3 = 0 \to t = -\frac{3}{4} \qquad 2(-\frac{3}{4})^2 + 3(-\frac{3}{4}) + 2 = \frac{7}{8}$

ج) $\sin^2 x + 4\cos^2 x$

$\sin^2 + \cos^2 = 1 \to y = \sin^2(x) + 4(1 - \sin^2(x)) = \sin^2 + 4 - 4\sin^2 = 4 - 3\sin^2 x$

$\sin^2 \in [0,1]$

$R = [1,4] \qquad \sin^2 = 0 \to 4 - 3\times 0 = 4$

$\sin^2 = 1 \to 4 - 3\times 1 = 1$

د) $\dfrac{2\cot(x)}{1+\cot(x)} \to \dfrac{2t}{1+t} \to 2 - \dfrac{2}{1+t} \to y = 2 - \dfrac{2}{1+t}$

$t \neq -1$

$↳ = \sin x \to R = [-1,1]$

$R = (-\infty, 2) \cup (2, +\infty)$

ه) $y = \sin^6 + \cos^6 \to (a+b)(a^2 - ab + b^2)$

$\underbrace{(\sin^2 + \cos^2)}_{1}((\sin^2(x))^2 - \sin^2(x)\cos^2 x + (\cos^2 x)^2)$

$y = \sin^4 x - \sin^2\cos^2 + \cos^4(x)$

$\sin^4 + \cos^4 = (\sin^2 + \cos^2)^2 - 2\sin^2\cos^2 =$

$1 - 2\sin^2\cos^2$

$y = (1 - 2\sin^2\cos^2) - \sin^2\cos^2 = 1 - 3\sin^2\cos^2$

$\sin^2\cos^2 = (\frac{\sin 2x}{2})^2 = \frac{\sin^2 2x}{4}$

$y = 1 - 3 \times \frac{\sin^2 2x}{4} = 1 - \frac{3}{4}\sin^2 2x$

$\sin^2 2x \in [0,1]$

$\sin^2 2x = 1$

$y_{min} = 1 - \frac{3}{4} = \frac{1}{4}$

$\sin^2 2x = 0$

$y_{max} = 1$

$R = [\frac{1}{4}, 1]$

درسنامه

$1{,}75$ $9)$

ب) $y = \sin^8 + \cos^8$

$a^4 + b^4 = (a^2+b^2)^2 - 2(ab)^2$

$\sin^8(x) + \cos^8(x) = (\sin^4(x))^2 + (\cos^4(x))^2 = (\sin^4(x) + \cos^4(x))^2 - 2\sin^4(x)\cos^4(x)$

$\sin^4 + \cos^4 = 1 - 2\sin^2\cos^2$

$y = (1 - 2\sin^2(x)\cos^2(x))^2 - 2(\sin^2(x)\cos^2(x))^2$

$\sin^2\cos^2 \leq \frac{1}{4}$

$y = (1-2t)^2 - 2t^2 = 1 - 4t + 4t^2 - 2t^2 = 1 - 4t + 2t^2$

$(\frac{1}{4})^2 - 4 \times \frac{1}{4} + 1 = 2(\frac{1}{16}) - \cancel{1+1} = \frac{1}{8}$

$R = \cancel{[\frac{1}{8}, 1]}$ $R = \{0, 1\}$

$4)$

$y = \frac{2x^2 + 7x - 4}{x+4} \longrightarrow R = \mathbb{R}$ $\frac{\cancel{(x+4)}(2x-1)}{\cancel{(x+4)}}$

$x \neq -4$

$15)$

$x \neq -4 \rightarrow R = \mathbb{R} - \{-9\}$

1

$y = \frac{x^3 - 1}{x - 1}$ $x \neq 1$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$x^3 - 1 = (x-1)(x^2 + x + 1)$

$\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}} \longrightarrow -\frac{b}{2a} = \frac{-1}{2}$

$R = [\frac{3}{4}, +\infty)$ $(-\frac{1}{2})^2 + -\frac{1}{2} + 1 = +\frac{1}{4} - \frac{2}{4} + \frac{4}{4}$

$\frac{3}{4}$