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الف) $y = x^3 - 3x^2 + 3x$

ب) $y = \frac{1}{x^2 - 1}$

ج) $y = x^2 - (x+1)$

الف) $y = x^3 - 3x^2 + 3x$ $R = \mathbb{R}$

$(x-1)^3 + 1 \rightarrow y-1 = (x-1)^3$
 $x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1$

ب) $y = \frac{1}{x^2 - 1}$

$x^2y - 1 = 0 \rightarrow x^2y - 1 = 0$

$x^2y = 1 \rightarrow x^2 = \frac{1}{y}$

$y^2 + y \gg 0$

$y(y+1) \gg 0$

$\frac{-1}{+} \quad \frac{0}{-} \quad \frac{+}{+}$

$R_f = (-\infty, -1] \cup [0, +\infty)$

الف) $y = x^2 - (x+1)$

ext $\left| \frac{-b}{2a} = \frac{1}{2} = \frac{1}{2} \right.$

$f = 1 + 1 = 2$
 min

$R_f = [2, +\infty)$

$$b) y = -x^2 + 4x + 12$$

$$x_{\max} = \frac{-4}{-2} = 2$$

$$y_{\max} = -9 + 18 + 12 = 12$$

$$R_f = (-\infty, 12]$$

$$2) y = \sqrt{x^2 - 6x + 9}$$

$$x_{\min} = \frac{6}{2} = 3$$

$$y_{\min} = 3 - 9 = -6$$

$$R_f = \sqrt{[-6, +\infty)} = [0, +\infty)$$

$$3) y = \sqrt{9x - x^2}$$

$$x_{\max} = \frac{-4}{-2} = 2$$

$$y_{\max} = 18 - 9 = 9$$

$$R_f = \sqrt{(-\infty, 9]} = [0, 3]$$

$$y = x^2 - 2x^2 + 12x + 1$$

$$R_f = \mathbb{R}$$

5) 4)

$$y = x^2 - 4x^2 + 12x + 1$$

$$R_f = \mathbb{R}$$

$$y = \sqrt{x^2 - 4x^2 + 12x + 1}$$

$$R_f = \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

$$y = (x^2 - 4x^2 + 12x + 1)^2$$

$$R_f = ((-\infty, +\infty))^2 = [0, +\infty)$$



الف) $y = \frac{n+1}{n-2}$

$R_f = \mathbb{R} - \left\{\frac{2}{a}\right\}$

⑤

$R_f = \mathbb{R} - \{2\}$

ب) $y = \frac{n+1}{n+1}$

$R_f = \mathbb{R} - \{1\}$

الف) $y = \sqrt{\frac{n+2}{n+1}}$

⑤

$R_f = \sqrt{\mathbb{R} - \{1\}} \rightarrow R_f = [0, +\infty) - \{\sqrt{1}\}$

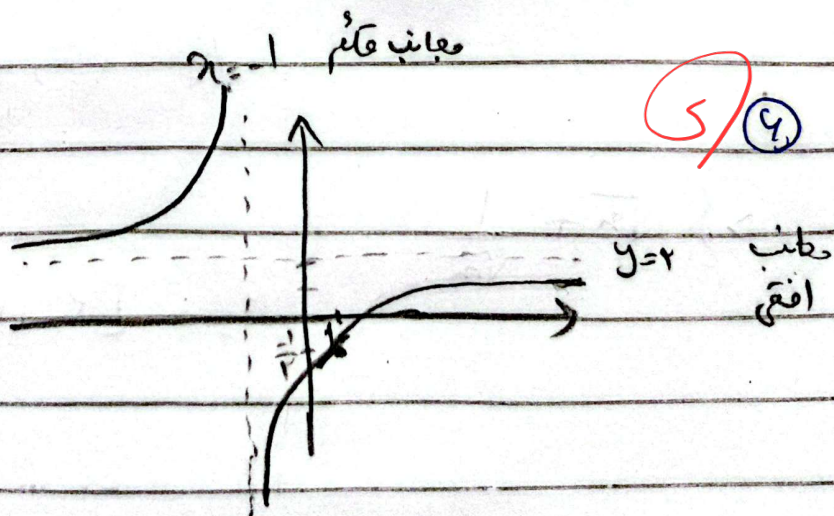
ب) $y = \sqrt{\frac{n+1}{1-n}}$

$R_f = \sqrt{\mathbb{R} - \{-1\}} \rightarrow R_f = [0, +\infty)$

الف) $y = \frac{n-3}{n+1}$

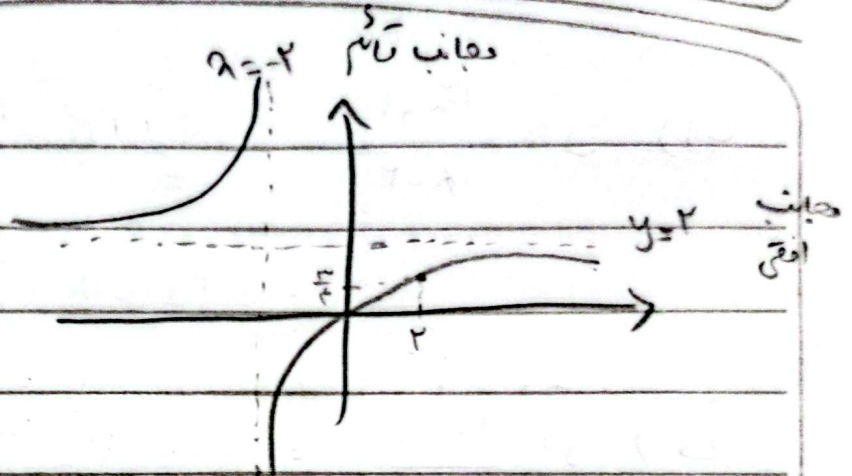
⑤

$n=1 \rightarrow y = \frac{-1}{2}$



ب) $y = \frac{x-1}{x+2}$

$x=2 \rightarrow y = \frac{1}{4}$



الف) $y = \sin x + \frac{1}{\sin x}$

$R_f = (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3}$

$R_f = (-\infty, -1] \cup [1, +\infty)$

2) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}}$

$R_f = (-\infty, -1] \cup [1, +\infty)$

> $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$R_f = [1, +\infty)$

الف) $y = x^2 + \frac{1}{x^2+3} + 3 - 3 =$

⑤ ①

$x^2 + 3 + \frac{1}{x^2+3} - 3$

سؤال $3 + \frac{1}{3} = \frac{10}{3} \rightarrow R_f = \left[\frac{10}{3}, +\infty \right)$

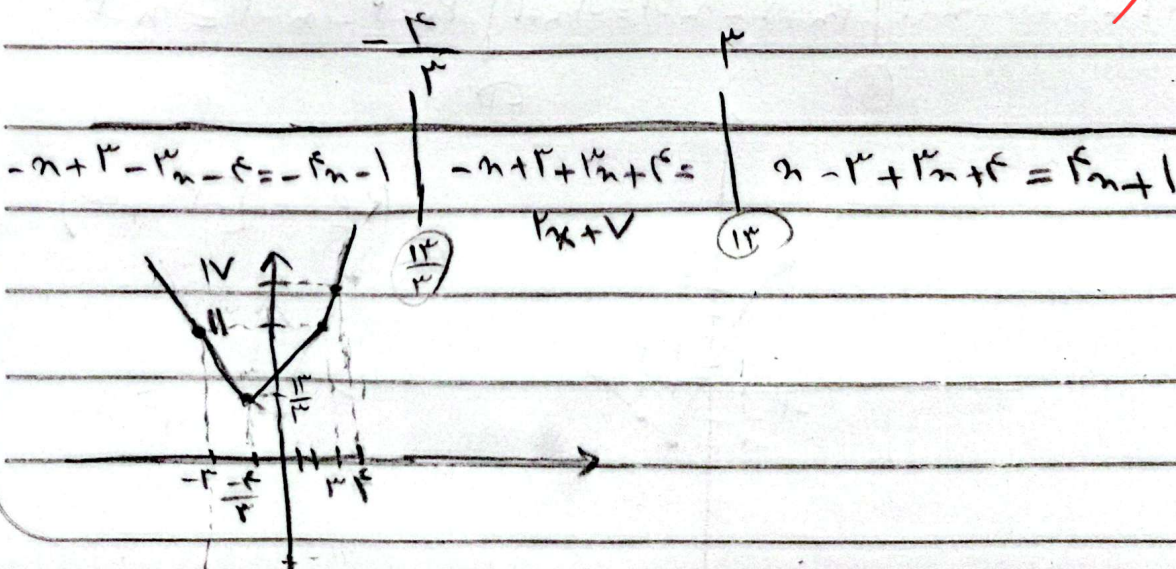
$y = x^2 + \frac{1}{x^2+3} \Rightarrow R_f = \left[\frac{1}{3}, +\infty \right)$

ب) $y = \frac{x^2+a}{\sqrt{x^2+1}} = \frac{x^2+1+1}{\sqrt{x^2+1}} = \sqrt{x^2+1} + \frac{1}{\sqrt{x^2+1}}$

سؤال $= \sqrt{1} + \frac{1}{\sqrt{1}} = 1 + 1 = \frac{2}{1}$

$R_f = \left[\frac{2}{1}, +\infty \right)$

$y = |x-3| + |x+3|$ ⑤ ②

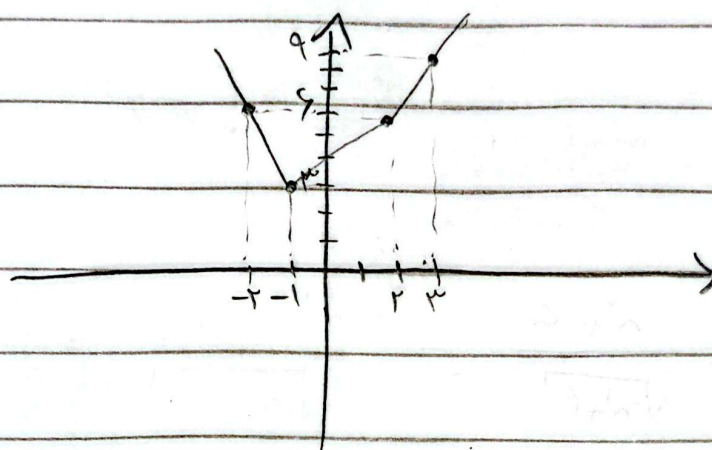




الف) $y = |x-2| + |x+2|$

١٥) ٥

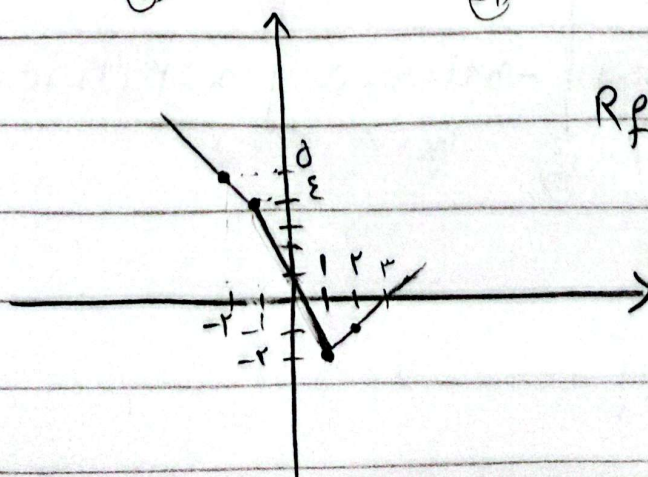
$$\begin{array}{c|c|c} -x+2-x-2=-4 & -x+2+x+2=x+4 & x-2+x+2=x \\ \hline \textcircled{2} & & \textcircled{2} \end{array}$$



$R_f = [2, +\infty)$

ب) $y = |x-2| - |x+1|$

$$\begin{array}{c|c|c} -x+2+x+1=3 & -x+2-x-1=-x+1 & x-2-x-1=-x-3 \\ \hline \textcircled{2} & & \textcircled{-2} \end{array}$$



$R_f = [-2, +\infty)$