

الف) $y = x^2 - 2x + 1 + 1 \Rightarrow y = (x-1)^2 + 1$



$R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow yx^2 - 2yx - 1 = 0 \Rightarrow \Delta \Rightarrow \varepsilon y^2 - \varepsilon(y)(-1) = \varepsilon y^2 + \varepsilon y$
 $\Rightarrow y \pm \sqrt{\varepsilon y^2 + \varepsilon y} \Rightarrow \varepsilon y^2 + \varepsilon y \geq 0 \Rightarrow \varepsilon y(y+1) \geq 0$

الف) $x^2 - \varepsilon x + 1$ $\Delta \Rightarrow 14 - \varepsilon(1)(1) = -2\varepsilon \Rightarrow \frac{-\Delta}{\varepsilon x} = \frac{2\varepsilon}{\varepsilon} = 2$
 $R_f = [4, +\infty)$

ب) $-x^2 + 4x + 3$ $\Delta = 14 - \varepsilon(4)(3) = 34 + 12 = 46$ $\frac{-\Delta}{\varepsilon x} \Rightarrow \frac{-46}{\varepsilon} = -12$
 $R_f = (-\infty, 12]$

ج) $\sqrt{x^2 - \varepsilon x - 3}$ $\Delta = 14 - \varepsilon(1)(-3) = 14 + 12 = 26 \Rightarrow \frac{-26}{\varepsilon} = -10$
 $R_f = [-10, +\infty)$

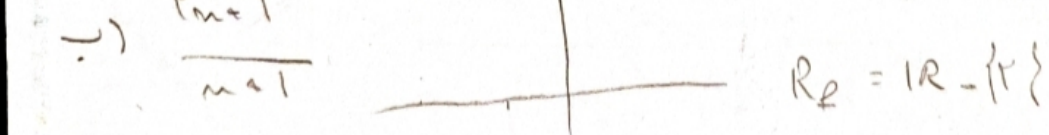
د) $\sqrt{4x - x^2}$ $\frac{4x}{x} = 4$ $R_f = (-\infty, 4]$

الف) $y = x^2 - 2x + 1$ $R_f = \mathbb{R}$

ب) $y = x^2 - 4x + 3$ $R_f = \mathbb{R}$

ج) $y = \sqrt{x^2 - 4x + 3}$ $R_f = [0, +\infty)$

د) $y = (x^2 - \varepsilon x + 1)^2$



$(y = \sqrt{\frac{x+1}{x-1}}) \Rightarrow y^2 - y^2 = x+1 \Rightarrow y^2 - x = y^2 + 1 \Rightarrow x(y^2 - 1) = y^2 + 1$
 $\frac{-\sqrt{x}}{+1} \frac{\sqrt{x}}{-1} \frac{y^2 + 1}{y^2 - 1}$

$(y = \sqrt{\frac{x+1}{x-1}})^2 \Rightarrow y^2 - y^2 = x+1$
 $R_f = (-\infty, -1] \cup (\frac{1}{2}, +\infty)$

$y = \frac{r_m - r}{n+1}$

$\rightarrow \frac{r_m - 1}{n+r}$

$y = \sin x + \frac{1}{\sin x} \quad R_f = (-\infty, -r] \cup [r, +\infty)$

$y = \frac{x^r + 1}{x^r} = x^r + \frac{1}{x^r} \quad R_f = (-\infty, -r] \cup [r, +\infty)$

$y = \frac{\sqrt[r]{x^r + 1}}{\sqrt[r]{x}} = \sqrt[r]{\frac{x^r}{x}} + \frac{1}{\sqrt[r]{x}} \quad R_f = (-\infty, -r] \cup [r, +\infty)$

$y = \sqrt{x} + \frac{1}{\sqrt{x}} \quad R_f = [r, +\infty)$

$y = x^r + \frac{1}{x^r + r} = x^r + \frac{1}{x^r + r} + r - r \quad R_f = [\frac{1}{r}, +\infty)$

$\frac{x^r + \infty}{\sqrt{x^r + \varepsilon}} = \frac{x^r + \varepsilon + 1}{\sqrt{x^r + \varepsilon}} = \sqrt{x^r + \varepsilon} + \frac{1}{\sqrt{x^r + \varepsilon}} \quad R_f = (\frac{\sqrt{\varepsilon}}{\varepsilon}, +\infty)$

$\mu_{\min} = \sqrt{\varepsilon} + \frac{1}{\sqrt{\varepsilon}} \quad \frac{\varepsilon + 1}{\sqrt{\varepsilon}} = \frac{\Delta}{\sqrt{\varepsilon}} = \frac{\Delta}{\sqrt{\varepsilon}}$

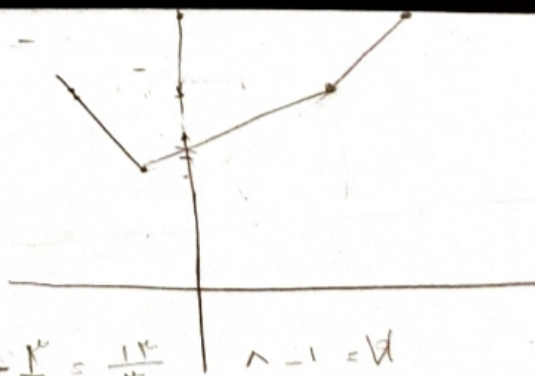
$y = |x - r| + |x_m + \varepsilon|$

$-\frac{\varepsilon}{r} \quad r$

$-\varepsilon x - 1 \quad r_m + v \quad r_m + 1$

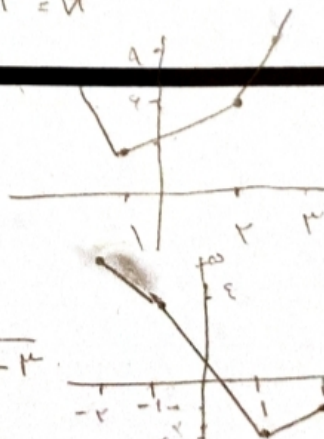
$-\frac{\varepsilon}{r} x + v = -\frac{\varepsilon}{r} + \frac{r}{r} = \frac{1}{r}$

$-\varepsilon x - \frac{\varepsilon}{r} - 1 = \frac{1}{r} - \frac{r}{r} = \frac{1}{r} \quad \wedge -1 = v$



$y = |x - r| + |x_m + r|$

$-\frac{1}{r_m} \quad n + \varepsilon \quad r$



$R_f = [r, +\infty)$

$y = |r_m - r| - |n + 1|$

$r_m - r - n - 1 = x - r$

$-r_m + r - n - 1 = r_m + r + n + 1 = -n + r$

