

الف)  $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{x^2-2x+1-x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x+1}{x(x-1)} \geq 0$

جدول علامت:

|   |               |   |
|---|---------------|---|
| + | +             | - |
| 0 | $\frac{1}{2}$ | 1 |
| + | -             | + |

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب)  $\frac{-x-2}{x(x+1)} \Rightarrow x(x+1) \neq 0 \Rightarrow x \notin \{0, -1\}$  ①

جواب:  $\mathbb{R} - \{0, -1\}$

②  $(x-1)(x-2) \neq 0, x+1 \neq 0 \Rightarrow x \notin \{1, 2, -1\}$

الف) عبارت اول در  $[-2, -\infty)$  نامنفی و عبارت دوم در  $[\frac{1}{3}, -\infty)$  نامنفی است  $\Leftarrow$  یا هر دو عبارت بابینا منفی باشند یا نامنفی  $\Leftarrow$

$D_f = (-\infty, -2] \cup [\frac{1}{3}, +\infty)$

ب)  $x-1 \geq 0 \Rightarrow x \geq 1$  ①,  $\sqrt{y+1} = 3 - \varepsilon \sqrt{x-1} \Rightarrow 3 - \varepsilon \sqrt{x-1} \geq 0 \Rightarrow x-1 \leq 1 \Rightarrow x \leq 2$  ②

$D_f = ① \cap ② = [1, 2]$

$\log_\varepsilon(x^2-x-2) \rightarrow x^2-x-2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow (-\infty, -1) \cup (2, +\infty)$  ①

$\sqrt{x^2-1} \rightarrow x^2-1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \leq -1$  ②

①  $\cap$  ②  $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$-x^2+ax+3 \geq 0 \xrightarrow{\text{جواب منفی}} -x^2-2x+3=0 \Rightarrow a = -\frac{1}{x}$

$-(x^2+\frac{1}{x}x-3) \geq 0 \rightarrow -(x+2)(x-\frac{3}{x}) \geq 0 \Rightarrow \frac{-2 \quad 3}{-\frac{1}{x} \quad + \quad 1} \Rightarrow D_f = [-2, \frac{3}{x}]$

$b = \frac{3}{x}$

جواب:  $a+b = \frac{3}{x} - \frac{1}{x} = \frac{2}{x} = 1$

$g(x) = \sqrt{f(x)-x} \Rightarrow f(x)-x \geq 0 \Rightarrow f(x) \geq x \Rightarrow 3x-2 \geq x \Rightarrow x \geq 1$  ①

$2x+3 \geq x \Rightarrow x \geq -3$  ②

جواب: ①  $\cap$  ②  $\Rightarrow [1, +\infty)$

$$f(a) = \sqrt{a} + \sqrt{1}, f(-1) = \sqrt{a} - \varepsilon$$

$$\hookrightarrow \sqrt{1}(\sqrt{a} + \sqrt{1}) = \sqrt{a} + \varepsilon + a \Rightarrow 10a = -11 \Rightarrow \boxed{a = -1, 1 = -\frac{9}{a}}$$

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~~$$\frac{1}{\sqrt{1-\sqrt{3}}} + \frac{1}{\sqrt{1+\sqrt{3}}} + 1+1 = \frac{1(1-\sqrt{3})}{\sqrt{1-\sqrt{3}}} + \frac{1(1+\sqrt{3})}{\sqrt{1+\sqrt{3}}} + \varepsilon = \frac{\varepsilon\sqrt{4} + 1\sqrt{4}}{\varepsilon} = \frac{12\sqrt{4}}{\varepsilon} + \varepsilon$$~~

$$\frac{1-\sqrt{3}}{\sqrt{1-\sqrt{3}}} + \frac{1+\sqrt{3}}{\sqrt{1+\sqrt{3}}} + 1+1 = \frac{1(1-\sqrt{3})}{\sqrt{1-\sqrt{3}}} + \frac{1(1+\sqrt{3})}{\sqrt{1+\sqrt{3}}} + \varepsilon = \frac{\varepsilon\sqrt{4} + 1\sqrt{4}}{\varepsilon} = \frac{12\sqrt{4}}{\varepsilon} + \varepsilon$$

$\boxed{\text{جواب} = 12\sqrt{4} + \varepsilon}$

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$$\begin{aligned} \sqrt{f(x)} - \sqrt{f(-x)} &= \varepsilon x^r - x \xrightarrow{\times \sqrt{r}} \begin{cases} 4f(x) - 9f(-x) = 11x^r + 11x \\ -4f(-x) + 9f(x) = 11x^r - 11x \end{cases} \\ -\sqrt{f(x)} + \sqrt{f(-x)} &= \varepsilon x^r + x \xrightarrow{\times \sqrt{r}} \end{aligned}$$

$$\Rightarrow -4f(x) = 10x^r + x$$

$$\downarrow$$

$$f(x) = \frac{10x^r + x}{-4}$$

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$$\begin{aligned} \forall x=0 &\Rightarrow \sqrt{f(0)} = \sqrt{m-1} \Rightarrow f(0) = \frac{m-1}{1} \\ x=-1 &\Rightarrow 4f(0) = 10 + 10m \Rightarrow f(0) = \frac{10+10m}{4} \end{aligned}$$

$$\left\{ \begin{aligned} \frac{10+10m}{4} &= \frac{m-1}{1} \Rightarrow 10+10m = 4m-4 \\ &\hookrightarrow 6m = -14 \\ &\hookrightarrow \boxed{m = -\frac{7}{3}} \end{aligned} \right.$$

$$f(0) = \frac{10}{4} = 2.5$$

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$$\sqrt{f(-1)} = \frac{\sqrt{x^r - 11x + 11}}{x} \Rightarrow f(-1) = \frac{x^r - 11x + 11}{x^2} \Rightarrow \frac{1+11+11}{-1} = \boxed{-9}$$

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