دیانا عابدی — یازدهم دفتر ۸ — تکلیف شماره ۲۰:

۱) الف) $\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}\geq 0 \to \frac{(x-1)^2-x^2}{x(x-1)}\geq 0 \to \begin{cases}1-2x=0\to x=\frac{1}{2}\\ x(x-1)=0\to x=0,\ x=1\end{cases}$

$\begin{array}{c|c|c|c}0 & \frac{1}{2} & 1 & \\ - \quad + & - & + & \end{array}$

$D=(0,\frac{1}{2}]\cup(1,+\infty)$

ب) $y=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}=\dfrac{\frac{x-2x-2}{x(x+1)}}{\frac{3x+6+x-1}{(x-1)(x+2)}}=\dfrac{\frac{-x-2}{x(x+1)}}{\frac{4x+5}{(x-1)(x+2)}}\to$

$\begin{cases}x(x+1)\neq 0\\ x\neq 0\\ x\neq -1\end{cases}$ $\quad\begin{cases}4x+5\neq 0\\ x\neq -\frac{5}{4}\end{cases}$ $\quad\begin{cases}(x-1)\neq 0\to x\neq 1\\ x\neq -2\end{cases}$ $\quad D=R-\{-1,0,-\frac{5}{4},1,-2\}$

۲) الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)}\geq 0$

$(\frac{1}{3})^x-9=0\to(\frac{1}{3})^x=9\to 3^{-x}=3^2\to x=-2$

$32=4^x\to x=\frac{5}{2}$ $\quad\begin{array}{c}-2 \qquad \frac{5}{2}\\ + \quad - \quad +\end{array}$ $\quad D=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

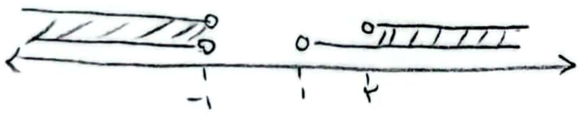
ب) $\underbrace{\sqrt[4]{(x-1)}}_{x\geq 1}{}_{\geq 0}+\underbrace{\sqrt{y+1}}_{y\geq -1}{}_{\geq 0}=3 \to \underbrace{\sqrt{y+1}}_{\geq 0}=\underbrace{3-\sqrt[4]{x-1}}_{\text{باید}\ \geq 0}\geq 0\to$

$\sqrt[4]{x-1}\leq 3 \xrightarrow{\text{توان ۴}} x-1\leq 81\to x\leq 82 \to 1\leq x\leq 82\to D=[1,82]$

۳) $f(x)=\dfrac{\log_2^{(x^2-x-2)}}{\sqrt{x^2-1}+1\neq 0}$ ✓ $\quad x^2-x-2>0\to(x-2)(x+1)=0\to\begin{cases}x=-1\\ x=2\end{cases}$ $\quad\begin{array}{c}-1 \qquad 2\\ + \quad - \quad +\end{array}$

$(-\infty,-1)\cup(2,+\infty)$

$x^2-1>0\to x^2>1\to |x|>1\to(-\infty,-1)\cup(1,+\infty)$



$D=(-\infty,-1)\cup(2,+\infty)$

۴) $\sqrt{3+ax-x^2}\geq 0 \quad x=-2\to 3+a(-2)-(-2)^2=3-2a-4=0\to 2a=1\to a=\frac{1}{2}$

$3+ax-x^2 \xlongequal{a=\frac{1}{2}} -x^2+\frac{1}{2}x+3=0 \xrightarrow{\times 2} -2x^2+x+6=0$

$x^2\frac{+1x}{-2}\frac{-12}{-2}=0\to(x+\frac{4}{-2})(x-\frac{3}{-2})=0\to\begin{cases}x=-2\\ x=-\frac{3}{2}\end{cases}\to D=[-2,-\frac{3}{2}]$

$a+b=\frac{1}{2}+(-\frac{3}{2})=-1$

۵) $f(x)=\begin{cases}3x-2 & ; x\geq 1\\ 2x+3 & , x<1\end{cases}$ $\quad g(x)=\sqrt{\underbrace{f(x)-x}}\geq 0$

1) $x\geq 1\to f(x)-x=3x-2-x=2x-2\geq 0\to x\geq 1\to [1,+\infty)\to$ در این بازه همه مقادیر $x\geq$ در دامنه هستند

2) $x<1\to f(x)-x=2x+3-x\geq 0\to x\geq -3\to -3\leq x<1$

$D_g=[-3,1)\cup[1,+\infty)=[-3,+\infty)$

$$f(x)=\begin{cases}(a+1)(x+2) & ; x>1\\ 3a+2x & ; x\leq 1\end{cases}\qquad 2f(5)=f(-2)+a$$ (6

$$f(5)=7(a+1)$$

$$f(-2)=3a-4 \rightarrow 2\times 7(a+1)=3a-4+a \Rightarrow 14a+14=4a-4 \Rightarrow$$

$$10a=-18 \Rightarrow a=-\frac{18}{10}=\frac{-9}{5}$$

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \rightarrow f(2-\sqrt{3})+f(2+\sqrt{3})$$ (7

$$a=2-\sqrt{3}\ ,\ b=2+\sqrt{3}=\frac{2+\sqrt{3}}{2-\sqrt{3}}\times 2-\sqrt{3}=\frac{1}{2-\sqrt{3}} \rightarrow b=\frac{1}{a}$$

$$a\cdot b=(2-\sqrt{3})\times\left(\frac{1}{2-\sqrt{3}}\right)=1$$

$$f(2-\sqrt{3})+f(2+\sqrt{3})=f(a)+f(b)=\left(\sqrt{a}+\frac{1}{\sqrt{a}}+2\right)+\left(\sqrt{b}+\frac{1}{\sqrt{b}}+2\right)=$$

$$(\sqrt{a}+\sqrt{b})+\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)+4 \overset{b=\frac{1}{a}}{=} \sqrt{a}+\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{a}}+\sqrt{a}+4=$$

$$2\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)+4 \overset{*}{=}$$

$$\sqrt{a}=\sqrt{2-\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{2}$$

$$\sqrt{a}+\frac{1}{\sqrt{a}}=\frac{\sqrt{6}-\cancel{\sqrt{2}}}{2}+\frac{\sqrt{6}+\cancel{\sqrt{2}}}{2}=\frac{\cancel{2}\sqrt{6}}{\cancel{2}}=\sqrt{6} \rightarrow$$

$$f(2-\sqrt{3})+f(2+\sqrt{3}) \overset{*}{=} 2\sqrt{6}+4$$

$$2f(x)-3f(-x)=4x^2-x \quad (**)$$ (8

$$x\rightarrow -x \Rightarrow 2f(-x)-3f(x)=4x^2+x$$

$$(+)\begin{cases}2f(x)-3f(-x)=4x^2-\cancel{x}\\ -3f(x)+2f(-x)=4x^2+\cancel{x}\end{cases}$$

$$-f(x)-f(-x)=8x^2 \rightarrow -f(x)-8x^2=f(-x) \quad *$$

$$\xrightarrow{(**)} 2f(x)-3\left(-f(x)-8x^2\right)=4x^2-x \Rightarrow 2f(x)+3f(x)+24x^2=4x^2-x$$

$$5f(x)=-20x^2-x \rightarrow f(x)=-4x^2-\frac{1}{5}x$$

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$ (9

$$\xrightarrow{x=0} 2f(0)=3m-1 \rightarrow f(0)=\frac{3m-1}{2} \quad (1)$$

$$\xrightarrow{x=-2} 6f(0)=16+2m+3m-1 \rightarrow 6f(0)=15+5m \rightarrow f(0)=\frac{15+5m}{6} \quad (2)$$

$$(1)=(2) \quad \frac{3m-1}{\cancel{2}_{1}}=\frac{15+5m}{\cancel{6}_{3}} \rightarrow 9m-3=15+5m \rightarrow 4m=18 \rightarrow m=\frac{18}{4}=\frac{9}{2}$$

۱۰)

$$f(x)+f\left(\frac{1}{x}\right)=\frac{۳x^{۲}-۱۲x+۳}{x}$$

$$\left.\begin{aligned} f(x)&=ax+b \\ f\left(\frac{1}{x}\right)&=\frac{a}{x}+b \end{aligned}\right\} \longrightarrow f(x)+f\left(\frac{1}{x}\right)=ax+b+a\left(\frac{1}{x}\right)+b=\frac{۳x^{۲}-۱۲x+۳}{x}$$

$$\Rightarrow \frac{ax^{۲}+bx+a+bx}{x}=\frac{ax^{۲}+۲bx+a}{\cancel{x}}=\frac{۳x^{۲}-۱۲x+۳}{\cancel{x}} \Rightarrow a=۳$$

$$۲b=-۱۲\rightarrow b=-۶$$

$$f(x)=۳x-۶ \Rightarrow f(-۱)=۳(-۱)-۶=-۹$$