نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره ...۴... کلاس ...یازدهم ریاضی A...

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الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \to \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{x^2-2x+1-x^2}{x^2-x}=\frac{1-2x}{x(x-1)}$

	0		$\frac12$		1	
$+$	∄	$-$	0	$+$	∄	$-$

$\boxed{D_f=(-\infty,0)\cup[\frac12,1)}$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \to x\neq 0,\ x\neq -1$

$\to \frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{3x+6+x-1}{(x-1)(x+2)}\neq 0 \to \frac{4x+5}{(x-1)(x+2)}\neq 0 \to x=-\frac54$

$\to x=+1,\ x=-2$

$\boxed{D_f=\mathbb{R}-\{-2,0,1,-\frac54,-1\}}$

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الف) $f(x)=\sqrt{((\frac13)^x-9)(32-4^x)} \to ((\frac13)^x-9)(32-4^x)\geq 0$

$x=-2 \qquad 2^5-2^{2x}=0 \to x=2.5$

	-2		2.5	
$+$	0	$-$	0	$+$

$\boxed{D_f=(-\infty,-2]\cup[2.5,+\infty)}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \to \sqrt{y+1}=3-\sqrt[4]{x-1} \to x-1\geq 0 \to x\geq 1$ (I) $\quad 3-\sqrt[4]{x-1}\geq 0 \to \sqrt[4]{x-1}\leq 3$

$x-1\leq 81 \to x\leq 82$ (II)

$\xrightarrow{I\cap II} \boxed{D_f=[1,82]}$

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$f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \to \log_4^{(x^2-x-2)} \to x^2-x-2>0 \to (x+1)(x-2)>0$

	-1		2	
$+$	0	$-$	0	$+$

$\to D_f=(-\infty,-1)\cup(2,+\infty)$ (I)

$\to x^2-1\geq 0 \to x^2\geq 1 \to x\geq 1,\ x\leq -1 \to D_f=(-\infty,-1]\cup[1,+\infty)$ (II)

$\xrightarrow{I\cap II} \boxed{D_f=(-\infty,-1)\cup(2,+\infty)}$

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$\sqrt{3+ax-x^2} \to [-2,b) \xrightarrow{\text{به ازای } x=-2 \text{ عبارت صفر}} 3-2a-4=0 \to -1-2a=0 \to 2a=-1 \Rightarrow a=-\frac12$

$\xrightarrow{a=-\frac12} -x^2-\frac12x+3 \to \Delta=\frac14-4(-1)(3)=\frac{49}{4} \to x=\dfrac{-(-\frac12)\pm\sqrt{\frac{49}{4}}}{-2} \to x_1=-2,\ x_2=\frac32$

$\Rightarrow b=\frac32 \qquad a+b=\frac32-\frac12=\frac22=1$

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$f(x)=\begin{cases}3x-2, & x\geq 1\\ 2x+3, & x<1\end{cases} \qquad g(x)=\sqrt{f(x)-x} \to f(x)-x\geq 0 \to f(x)\geq x$

$\Rightarrow 3x-2\geq x \to 2x-2\geq 0 \to x-1\geq 0$

$x\geq 1$ همواره برقرار (I)

$\Rightarrow 2x+3\geq x \to x+3\geq 0 \to x\geq -3$ (II)

$\xrightarrow{I\cap II} \boxed{D_f=[-3,+\infty)}$

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$$f(x)=\begin{cases}(a+1)(x+2) & ,\ x>1\\ 2a+2x & ,\ x\leq 1\end{cases}$$

$$2f(5)=f(-2)+a$$

$$f(5)=(a+1)(7)\to f(5)=7a+7\to 2f(5)=14a+14$$

$$f(-2)=2a-4$$

$$14a+14=2a-4+a\Rightarrow 14a+14=4a-4\to 10a=-18\Rightarrow \boxed{a=-1/8}$$

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$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \qquad f(2-\sqrt{3})+f(2+\sqrt{3})=f(2+\sqrt{3})+f\left(\frac{1}{2+\sqrt{3}}\right)$$

$$(2+\sqrt{3}=t)\to f(t)+f\left(\frac{1}{t}\right)=2\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)+4\to 2\left(\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}\right)+4$$

$$2\underbrace{\left(\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}\right)}_{K}+4\to K^2=(2-\sqrt{3})(2+\sqrt{3})+2\sqrt{(2+\sqrt{3})(2-\sqrt{3})}=6\to K=\sqrt{6}$$

$$f(2-\sqrt{3})+f(2+\sqrt{3})=2K+4\to \boxed{2\sqrt{6}+4}$$

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$$2f(x)-3f(-x)=4x^2-x \quad (\text{ضرب در } 2) \qquad \to 2f(-x)-3f(x)=4x^2-x \quad (\text{ضرب در } 3)$$

$$\begin{cases}\cancel{6f(-x)}-9f(x)=12x^2-3x\\ 4f(x)-\cancel{6f(-x)}=8x^2-2x\end{cases}$$

$$-5f(x)=20x^2+x\to \boxed{f(x)=-\frac{20x^2+x}{5}}$$

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$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$

$$\xrightarrow{x=0} 2f(0)=3m-1$$

$$\xrightarrow{x=-2} 6f(0)=16+2m+3m-1\to 6f(0)=5m+15$$

$$\begin{cases}6f(0)=5m+15\\ (2f(0)=3m-1)\times 3\end{cases}$$

$$-4m+18=0\to 4m=18$$

$$m=\frac{18}{4}=\frac{9}{2}$$

$$\xrightarrow{f(0)} 2f(0)=3\left(\frac{9}{2}\right)-1=\frac{25}{2}\to \boxed{f(0)=\frac{25}{4}}$$

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$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}=3x-12+\frac{3}{x}$$

$$\left.\begin{aligned}f(x)&=ax+b\\ f\left(\tfrac{1}{x}\right)&=a\tfrac{1}{x}+b\end{aligned}\right\} f(x)+f\left(\frac{1}{x}\right)=a\left(x+\frac{1}{x}\right)+2b=ax+\frac{a}{x}+2b$$

$$3x-12+\frac{3}{x}=ax+\frac{a}{x}+2b\to a=3,\ b=-6$$

$$\Rightarrow f(x)=3x-6\to f(-1)=-3-6=-9$$