تکلیف شماره ۴ — * ترنم کسائیان

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الف)

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow x \neq 0,\ x \neq 1 \quad ①$$

$$② \quad \frac{x-1}{x} \geq \frac{x}{x-1} \Rightarrow \frac{1-2x}{x^2-x} \geq 0 \rightarrow \begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & \circ & - & \bullet & + & \circ & - \end{array} \Rightarrow$$

$$① \cap ② \rightarrow D_f = (-\infty, 0) \cup [\tfrac{1}{2}, 1)$$

ب)

$$f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} \Rightarrow x \neq -2,\ x \neq -1,\ x \neq 0,\ x \neq 1 \quad ①$$

$$② \quad \frac{3}{x-1} \neq \frac{1}{x+2} \Rightarrow \frac{4x+5}{(x-1)(x+2)} \neq 0 \rightarrow x \neq -2,\ x \neq -\frac{5}{4},\ x \neq 1$$

$$① \cap ② \rightarrow D_f = \mathbb{R} - \{-2, -\tfrac{5}{4}, -1, 0, 1\}$$

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الف)

$$f(x) = \sqrt{\left(\left(\tfrac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)} \Rightarrow \left(\left(\tfrac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right) \geq 0$$

$$\Rightarrow \begin{array}{ccccc} & -2 = \log_{\frac{1}{3}} 9 & & \frac{5}{2} = \log_4 32 & \\ + & \bullet & - & \bullet & + \end{array} \Rightarrow D_f = (-\infty, -2] \cup [\tfrac{5}{2}, +\infty)$$

ب)

$$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \rightarrow ① \quad x-1 \geq 0 \Rightarrow x \geq 1$$

$$② \quad \sqrt[4]{x-1} \leq 3 \Rightarrow x-1 \leq 81 \rightarrow x \leq 82 \Rightarrow ① \cap ② \rightarrow D_f = [1, 82]$$

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صورت همواره مثبت ،

$$x^2 - x - 2 > 0 \rightarrow \begin{array}{ccccc} & -1 & & 2 & \\ + & \circ & - & \circ & + \end{array} \quad ①$$

$$② \quad x^2 - 1 \geq 0 \Rightarrow x \geq 1,\ x \leq -1 \longrightarrow ① \cap ② \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

۴

$$x = -2 \longrightarrow 3 - 2a - 4 = 0 \longrightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$$

$$\longrightarrow -x^2 - \frac{1}{2}x + 3 \rightarrow x = \frac{b^2 \pm \sqrt{\Delta}}{2a}, \quad \sqrt{b^2 - 4ac} = \sqrt{\frac{49}{4}} \rightarrow x = \frac{\frac{1}{2} \pm \frac{7}{2}}{-2} \Rightarrow x = -2,\ x = \frac{3}{2}$$

$$\Rightarrow b = \frac{3}{2} \Rightarrow \boxed{a + b = 1}$$

(۵)

$f(x)-x \geq 0 \Rightarrow f(x) \geq x \xrightarrow{x \geq 1} 3x-2 \geq x \Rightarrow x \geq 1$ برقرار است

$\xrightarrow{} x<1 \Rightarrow 2x+3 \geq x \Rightarrow x \geq -3$

$\Rightarrow D_f = [-3, +\infty)$

(۶)

$f(5) = (a+1)\times 7 \Rightarrow 2f(5) = 14a+14 \xrightarrow{2f(5)=f(-2)+a}$

$f(-2)+a = 14a+14 \quad , \quad f(-2) = 3a-4 \Rightarrow$

$4a-4 = 14a+14 \Rightarrow 10a = -18 \Rightarrow a = -\frac{18}{10} = -1/8$

(۷)

$\frac{1}{2+\sqrt{3}} = 2-\sqrt{3} \Rightarrow f(a)+f(\frac{1}{a}) = 2\times(\sqrt{a}+\frac{1}{\sqrt{a}})+4$

$2+\sqrt{3} \rightarrow a$

$\Rightarrow 2(\underbrace{\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}}_{\sqrt{6}})+4 = \boxed{2\sqrt{6}+4}$

(۸)

$2f(x)-3f(-x) = 4x^2-x$

$2f(-x)-3f(x) = 4x^2+x$

$\Rightarrow 4f(x)-6f(-x) = 8x^2-2x \quad + \quad 6f(-x)-9f(x) = 12x^2+3x$

$\Rightarrow -5f(x) = 20x^2+x \Rightarrow f(x) = \frac{20x^2+x}{-5}$

(۹)

$x=0 \Rightarrow 2f(0) = 3m-1 \xrightarrow{\times 3} 6f(0) = 9m-3$

$x=-2 \Rightarrow 6f(0) = 16+2m+3m-1 \Rightarrow 15+5m = 9m-3 \Rightarrow$

$4m = 18 \Rightarrow m = 4/5 \Rightarrow 2f(0) = \frac{27-2}{2} = \frac{25}{2} \Rightarrow$

$f(0) = \frac{25}{4}$

(۱۰)

$f(x)+f(\frac{1}{x}) = \frac{3x^2-12x+3}{x} \xrightarrow{x=-1} 2f(-1) = (3+12+3)-1$

$\Rightarrow 2f(-1) = -18 \Rightarrow \boxed{f(-1) = -9}$