«اصغر صادقی»

۱)

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \Rightarrow x\neq 0\ ,\ x\neq 1\ ,\ \frac{x-1}{x}-\frac{x}{x-1}\geq 0$

$\frac{x^2+1-2x-x^2}{x(x-1)} \to \frac{1-2x}{x(x-1)}\geq 0$

| | $0$ | $\frac{1}{2}$ | $1$ | |
|---|---|---|---|---|
| $+$ | | $-$ | $+$ | $-$ |

$D\to(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \Rightarrow x\neq -1\ ,\ x\neq 0\ ,\ x\neq 1\ ,\ x\neq -2$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{3x+6+x-1}{x^2+2x-x-2} \to \frac{4x+5}{x^2+x-2}\neq 0$

$4x+5\neq 0 \to 4x\neq -5\quad x\neq\frac{-5}{4}\qquad D=\mathbb{R}-\{-1,0,1,-2,-\frac{5}{4}\}$

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۲)

الف) $f(x)=\sqrt{\left[\left(\frac{1}{3}\right)^x-9\right](32-4^x)} \Rightarrow \left(\frac{1}{3}\right)^x-9=0 \to 3^{-x}=9 \to 3^2 \to x=-2$

$32-4^x=0 \to 4^x=32 \to x=\frac{5}{2}$

$32-4^x>0\ ,\ \left(\frac{1}{3}\right)^x-9>0$ if $x<-2$ $\xrightarrow{\text{حاصل ضرب}}$ $\square>0$

$32-4^x>0\ ,\ \left(\frac{1}{3}\right)^x-9<0$ if $-2<x<\frac{5}{2}$ $\xrightarrow{\text{حاصل ضرب}}$ $\square<0$ ناحق

$32-4^x<0\ ,\ \left(\frac{1}{3}\right)^x-9<0$ if $x>\frac{5}{2}$ $\xrightarrow{\text{حاصل ضرب}}$ $\square>0$

$D=(-\infty,-2]\cup[\frac{5}{2},\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \longrightarrow \sqrt[4]{x-1}+\sqrt{y+1}-3=0$

$x-1\geq 0\to x\geq 1\qquad y+1\geq 0\to y\geq -1\qquad D=\{(x,y)\in\mathbb{R}^2\mid x\geq 1,\ y\geq -1\}$

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۳)

$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow \sqrt{x^2-1}+1\neq 0 \to x<-1$ یا $x>1$ ①

$x^2-x-2>0\to(x-2)(x+1)>0\to x>2$ یا $x<-1$ ②

① ∩ ② $\longrightarrow D=(-\infty,-1)\cup(2,+\infty)$

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۴)

$\sqrt{3+ax-x^2} \Rightarrow D=[-2,b] \rightsquigarrow a+b=?$

کوچکترین مقدار ← به ازای آن عبارت صفر است

$3-2a-4=0\to -1=2a\to a=-\frac{1}{2}$

$3-\frac{1}{2}x-x^2 \to$ محور تقارن $=-\frac{1}{2}=\frac{-b}{a}\qquad b=-\frac{1}{2}-(-2)=\frac{3}{2}$

$a+b=-\frac{1}{2}+\frac{3}{2}=1$

«اصغر مارانی»

۵)

$$f(x)=\begin{cases} 3x-2 & : x\geq 1 \\ 2x+3 & , x\leq 1 \end{cases} \longrightarrow g(x)=\sqrt{f(x)-x} = ?$$

$\hookrightarrow f(x)-x\geq 0$

$x\geq 1 \to 3x-2-x\geq 0 \to 2x-2\geq 0$ ; $x\geq 1$

$x<1 \to 2x+3-x\geq 0 \to x+3\geq 0$ ; $x\geq -3$

$D=[-3,1)\cup[1,+\infty) \longrightarrow x\geq -3$

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۶)

$$f(x)=\begin{cases} (a+1)(x+2) & : x>1 \\ 3a+2x & : x\leq 1 \end{cases} \quad , \; 2f(5)=f(-2)+a \qquad a=?$$

$2f(5)=f(-2)+a \longrightarrow f(5)\to (a+1)(7)=7a+7$

$f(-2)\to -4+3=-1$

$2(7a+7)=-1+a \longrightarrow 14a+14=-1+a \to 13a=-15 \to a=\frac{-15}{13}$

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۷)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \qquad f(2-\sqrt{3})+f(2+\sqrt{3})=?$$

$f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2=\frac{\sqrt{3}+1}{\sqrt{2}}+\frac{1}{\frac{\sqrt{3}+1}{\sqrt{2}}}+2$

$f(2+\sqrt{3})=\frac{\sqrt{3}+1}{\sqrt{2}}+\frac{\sqrt{2}}{\sqrt{3}+1}+2 \longrightarrow \frac{\sqrt{3}+1}{\sqrt{2}}+\frac{\sqrt{6}-\sqrt{2}}{2}+2 \to \frac{\sqrt{6}+\sqrt{2}}{2}+\frac{\sqrt{6}-\sqrt{2}}{2}+2$

$\longrightarrow \sqrt{6}+2$

$f(2-\sqrt{3}) \xrightarrow{\text{همین روش}} \sqrt{6}+2$

$f(2-\sqrt{3})+f(2+\sqrt{3})=\sqrt{6}+2+\sqrt{6}+2=2\sqrt{6}+4$

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۸)

$$2f(x)-3f(-x)=4x^2-x \qquad f(x)=?$$

$2f(x)-3f(-x)=4x^2-x \xrightarrow{\times 2} 4f(x)-\cancel{6f(-x)}=8x^2-2x$

$2f(-x)-3f(x)=4x^2+x \xrightarrow{\times 3} \cancel{6f(-x)}-9f(x)=12x^2+3x$

$-5f(x)=20x^2+x \to f(x)=\frac{20x^2+x}{-5} \longrightarrow f(x)=-4x^2-\frac{x}{5}$

" اکبر معارفی "

9)

$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1 \qquad f(0) = ?$

$f(x) = ax^2 + bx + c \rightarrow (ax^2+bx+c)(x+2) - 3x(a(x+2)^2 + b(x+2) + c) =$

$4x^2 - mx + 3m - 1 \rightarrow (ax^2+bx+c)(x+2) = ax^3 + 2ax^2 + bx^2 + 2bx +$

$cx + 2c = ax^3 + (2a+b)x^2 + (2b+c)x + 2c \cdot a(x+2)^2 = a(x^2+4x+4) =$

$ax^2 + 4ax + 4a \quad b(x+2) = bx + 2b$

$3x(a(x+2)^2 + b(x+2) + c) = 3x(ax^2 + 4ax + 4a + bx + 2b + c) =$

$3x(ax^2 + (4a+b)x + (4a+2b+c)) = 3ax^3 + 3(4a+b)x^2 + 3(4a+2b+c)x =$

$3ax^3 + (12a+3b)x^2 + (12a+6b+3c)x$

$(ax^3 + (2a+b)x^2 + (2b+c)x + 2c) - (3ax^3 + (12a+3b)x^2 + (12a+6b+3c)$

$x = 4x^2 - mx + 3m - 1$

ضریب $x^3$ $= 0 \rightarrow a - 3a = -2a = 0 \rightarrow a = 0 \rightarrow f(x) = bx + c$ $\quad f(0) = -2(0) + \frac{25}{4} = \frac{25}{4}$

با تکرار دوباره مراحل قبل $\rightarrow$ ضریب $x^2$ $\rightarrow b = -2 \qquad$ ضریب $x$ $\rightarrow m = 2c - 8 \rightarrow c = \frac{25}{4}$

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10)

$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \qquad f(-1) = ?$

$f(x) = ax + b \rightarrow ax + b + a\left(\frac{1}{x}\right) + b = ax + b + \frac{a}{x} + b = ax + \frac{a}{x} + 2b$

$\frac{3x^2 - 12x + 3}{x} = \frac{3x^2}{x} - \frac{12x}{x} + \frac{3}{x} = 3x - 12 + \frac{3}{x}$

$a = 3$
$b = -6$

$f(x) = 3x - 6 \rightarrow f(-1) = 3(-1) - 6 = -9$