آکادمی رسانی — پاسخنامه تشریحی شماره ۳ — یازدهم ریاضی A

۱۹٫۷۵

①

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \Rightarrow \frac{1-2x}{x^2-x}\geqslant 0 \Rightarrow$ $x(x-1)\neq 0$ ، $0, 1$

$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

۵

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} = \dfrac{\frac{x-2x-2}{x(x+1)}}{\frac{3x+6+x-1}{(x-1)(x+2)}}$

$x(x+1) \to 0, -1$

$3x+6+x-1 \to 4x+5\neq 0 \Rightarrow x\neq \frac{-5}{4}$

$(x-1)(x+2) \to 1, -2$

$D_f=\mathbb{R}-\{0,\pm 1,-2,-\frac{5}{4}\}$

②

۲٫۵

الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)}$

$3^{-x}=9 \Rightarrow x=-2$ ، $32=4^x \Rightarrow \log_4^{32}=x$

$D_f=(-\infty,-2]\cup[\log_4^{32},+\infty)$

ب) $\sqrt{y+1}+\sqrt[4]{x-1}=3 \Rightarrow \sqrt{y+1}=-\sqrt[4]{x-1}+3 \Rightarrow -\sqrt[4]{x-1}+3\geqslant 0 \Rightarrow 3\geqslant\sqrt[4]{x-1}$

$x-1\geqslant 0 \Rightarrow x\geqslant 1$

$81\geqslant x-1 \Rightarrow 82\geqslant x \Rightarrow D_f=[1,82]$

۱٫۷۵

③

$f(x)=\dfrac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1}$ ، $(x-2)(x+1)$

$\sqrt{x^2-1}+1\neq 0 \Rightarrow$ همواره صحیح

$x^2-1\geqslant 0 \Rightarrow x^2\geqslant 1 \Rightarrow x\geqslant 1 , x\leqslant -1$

$D_f=(-\infty,-1)\cup(2,+\infty)$

۵

④

$\sqrt{-x^2+ax+3}$ ، $D_f=[-2,b]$ $\xRightarrow{x=-2}$ $-4-2a+3=0 \Rightarrow 2a=-1, a=-\frac{1}{2}$

$\sqrt{-x^2-\frac{1}{2}x+3} \Rightarrow x^2-\frac{1}{2}x+3=0 \Rightarrow x^2+\frac{1}{2}x-3=0 \Rightarrow 2x^2+x-6$

$\frac{-1\pm\sqrt{1+48}}{4}=$ $x=\frac{-8}{4}=-2$ ، $x=\frac{6}{4}=\frac{3}{2}=b$

$a+b=-\frac{1}{2}+\frac{3}{2}=1$

۵

⑤

$f(x)-(x)\geqslant 0 \Rightarrow f(x)\geqslant x$

if $x\geqslant 1 \Rightarrow f(x)=3x-2 \Rightarrow \checkmark$

if $x<1 \Rightarrow f(x)=2x+3 \Rightarrow 1>x\geqslant -3$

$D_f=[-3,+\infty)$

۵

6)

$f(5) = (a+1)(7) \Rightarrow 7a+7 \Rightarrow 2f(5) = 14a+14$

$f(-2) = 3a-4 \,,\; 2f(5) - a = f(-2) \Rightarrow 13a+14 = 3a-4 \Rightarrow a = -1.8$ 3

---

7)

$f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \Rightarrow \sqrt{2-\sqrt{3}} + \sqrt{\cancel{2+\sqrt{3}}} + 2$

$f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2$

$\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2 + \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 = 2\left(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}\right) + 4$ 5

$t^2 = 2-\sqrt{3} + 2+\sqrt{3} + 2(\sqrt{1}) = 6 \Rightarrow t = \sqrt{6} \Rightarrow \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} = \sqrt{6}$

$f(2-\sqrt{3}) + f(2+\sqrt{3}) = 2\sqrt{6} + 4$

---

8)

$2f(x) - 3f(-x) = 4x^2 - x$

$2f(-x) - 3f(x) = 4x^2 + x$

$\begin{cases} 4f(x) - 6f(-x) = 8x^2 - 2x \\ 6f(-x) - 9f(x) = 12x^2 + 3x \end{cases}$

5

$-5f(x) = 20x^2 + x \longrightarrow f(x) = -\frac{20x^2+x}{5}$

---

9)

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$\xrightarrow{x=0} 2f(0) = 3m - 1$

$\xrightarrow{x=-2} 9f(0) = 16 + \underbrace{2m + 3m}_{5m} - 1$

$\begin{cases} 10f(0) = 15m - 5 \\ 18f(0) = 45 + 15m \end{cases}$ 5

$8f(0) = 50 \Rightarrow f(0) = 6.25$

---

10)

$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \Rightarrow f(x) = ax + b$

$ax + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \Rightarrow$ 5 $\frac{ax^2 + 2bx + a}{x} \Rightarrow a = 3 \,,\; b = -6$

$f(-1) = -3 - 6 = -9$