ملیکا محمد حسینی

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(۱)

$$f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \rightarrow \frac{x-1}{x}-\frac{x}{x-1} \rightarrow \frac{(x-1)^2-x}{x(x-1)}=\frac{1-2x}{x(x-1)} \rightarrow x\neq 0,1$$

	0		$\frac{1}{2}$		1	
$+$		$-$		$+$		$-$

$$D=(-\infty,0)\cup\left[\tfrac{1}{2},1\right)$$

$$f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \rightarrow x\neq 1,-2,-1,0$$

$$\mathbb{R}-\left\{0,-1,1,-2,-\tfrac{5}{4}\right\}$$

$$\frac{3}{x-1}+\frac{1}{x+2} \rightarrow \frac{3(x+2)}{(x+2)(x-1)}+\frac{1(x-1)}{(x+2)(x-1)}=\frac{3x+6+x-1}{(x+2)(x-1)} \Rightarrow 4x+5=0 \rightarrow x\neq -\frac{5}{4}$$

(۲)

$$f(x)=\sqrt{\left[\left(\tfrac{1}{3}\right)^x-9\right](32-4^x)} \rightarrow (3^{-x}-9)(32-4^x)\geq 0$$

$$3^{-x}-9=0 \rightarrow 3^{-x}=3^2 \rightarrow -x=2 \rightarrow x=-2$$

$$32-4^x=0 \rightarrow 4^x=32 \rightarrow 2^{2x}=2^5 \rightarrow 2x=5 \rightarrow x=\frac{5}{2}$$

$$D=(-\infty,-2]\cup\left[\tfrac{5}{2},+\infty\right)$$

$$\sqrt[4]{x-1}+\sqrt{y+1}=3 \rightarrow x-1\geq 0 \rightarrow x\geq 1 \qquad y+1\geq 0 \rightarrow y\geq -1$$

$$\sqrt{y+1}=3 \rightarrow y=8 \qquad y \quad \sqrt[4]{x-1}=3 \rightarrow x=82$$

$$0\leq\sqrt[4]{x-1}\leq 3 \rightarrow 1\leq x\leq 82 \rightarrow D=[1,82]$$

(۳) دامنه‌ی تابع با ضابطه‌ی $f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$ را بدست آورید

$$x^2-1\geq 0 \rightarrow x^2\geq 1 \rightarrow x\geq 1 \qquad x\leq -1$$

$$\sqrt{x^2-1}+1\neq 0 \rightarrow \sqrt{x^2-1}\neq -1$$

$$x^2-x-2>0 \rightarrow x^2-x-2=(x+1)(x-2)>0$$

$x-2>0 \rightarrow x>2$ یا هردو +

$x+1<0 \rightarrow x<-1$ یا هردو منفی

$D_f=(-\infty,-1)\cup(2,+\infty) \leftarrow \cap$ $\quad x\geq 1$ یا $x\leq -1$ مخرج

$x>2$ یا $x<-1$ لگاریتم

(۴)

$$\sqrt{3+ax-x^2} \qquad [-2,b] \qquad a+b=? \qquad 3+ax-x^2\geq 0 \rightarrow x^2-ax-3\leq 0$$

$$x=\frac{a\pm\sqrt{a^2+12}}{2} \rightarrow x_{min}=\frac{a-\sqrt{a^2+12}}{2}=-2 \rightarrow a-\sqrt{a^2+12}=-4 \rightarrow \sqrt{a^2+12}=a+4$$

$$a^2+12=(a+4)^2 \rightarrow \cancel{a^2}+12=\cancel{a^2}+8a+16 \rightarrow 8a+4 \Rightarrow 8a=-4 \rightarrow \boxed{a=-\frac{1}{2}}$$

$$\frac{a+\sqrt{a^2+12}}{2}=\frac{-\frac{1}{2}+\sqrt{(-\frac{1}{2})^2+12}}{2}=\frac{-0.5+3.5}{2}=\boxed{\frac{3}{2}=1.5=b}$$

$$a+b=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=1$$

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$$f(x)=\begin{cases}3x-2 & ; x\geq 1\\ 2x+3 & ; x<1\end{cases}$$

$g(x)=\sqrt{f(x)-x} \rightarrow f(x)-x\geq 0 \rightarrow f(x)\geq x$

$3x-2-x=2x-2\geq 0 \rightarrow 2x\geq 2 \rightarrow x\geq 1$

$2x+3-x=x+3\geq 0 \rightarrow x\geq -3 \xrightarrow{\cap} D=[-3,+\infty)$

(۶)

$$f(x)=\begin{cases}(a+1)(x+2) & ; x\geq 1\\ 3a+2x & ; x\leq 1\end{cases}$$

$2f(5)=f(-2)+a \rightarrow a=?$

$f(5)=(a+1)(5+2)=7(a+1)=7a+7$

$f(-2)=3a+2(-2)=3a-4$

$2(7a+7)=3a-4+a \rightarrow 14a+14=3a-4+a \rightarrow 10a=-18 \rightarrow a=-\frac{18}{10} \rightarrow \boxed{a=-\frac{9}{5}}$

(۷)

$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \qquad f(\underbrace{2-\sqrt{3}}_{a})+f(\underbrace{2+\sqrt{3}}_{b})=? \rightarrow s=\sqrt{a} \;,\; t=\sqrt{b} \rightarrow t=\frac{1}{s}$

$ab=(2-\sqrt{3})\times(2+\sqrt{3})=4-3=1 \qquad a^2+b^2=6 \qquad s+t=s+\frac{1}{s}$

$f(a)+f(b)=(s+\frac{1}{s}+2)+(t+\frac{1}{t}+2)=(s+t)+(\frac{1}{s}+\frac{1}{t})+4$

$f(a)+f(b)=2(s+\frac{1}{s})+4 \qquad s^2=a\;,\; t^2=b$

$(s+t)^2=s^2+2\underbrace{st}_{1}+t^2=a+2+b \longrightarrow (2-\sqrt{3})+2+(2+\sqrt{3})=6$

$s+\frac{1}{s}=\sqrt{6} \qquad \boxed{f(2-\sqrt{3})+f(2+\sqrt{3})=4+2\sqrt{6}}$

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$2f(x)-3f(-x)=4x^2-x \longrightarrow \begin{cases}2f(-x)-3f(x)=4x^2+x\\ 2f(x)-3f(-x)=4x^2-x\end{cases}$

$\boxed{f(x)=-4x^2-\frac{x}{5}}$ ✓

$f(-x)=-4x^2+\frac{x}{5}$

(۹)

$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1 \qquad f(0)=?$

$(0+2)f(0)-\cancel{3(0)f(0+2)}=\cancel{4(0)^2}-\cancel{m(0)}+3m-1$

$2f(0)-0=3m-1 \rightarrow \boxed{f(0)=\frac{3m-1}{2}}$

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$f(x)+f(\frac{1}{x})=\frac{3x^2-12x+3}{x} \qquad f(-1)=?$

$(ax+b)+(a\frac{1}{x}+b)=ax+\frac{a}{x}+2$

$f(x)+f(\frac{1}{x})=3x-12+\frac{3}{x} \rightarrow a=3 \;,\; 2b=-12 \rightarrow b=-6$

پ: $f(x)=3x-6 \rightarrow f(-1)=3(-1)-6=-3-6=\boxed{-9}$

$x=-2 \rightarrow f(0)=\frac{15+5m}{4} \qquad \frac{3m-1}{2}=\frac{15+5m}{4} \rightarrow m=\frac{9}{2}$ (۹

$f(0)=\frac{25}{4}$