

$$f(u) = \begin{cases} 3u-2 & ; u \geq 1 \\ 3u+3 & ; u < 1 \end{cases}$$

$$g(u) = \sqrt{f(u)-u} \Rightarrow f(u)-u \geq 0 \Rightarrow \begin{cases} \text{for } u \geq 1 \xrightarrow{f(u)=3u-2} 3u-2-u \geq 0 \Rightarrow 2u \geq 2 \Rightarrow u \geq 1 \text{ (I)} \\ \text{for } u < 1 \xrightarrow{f(u)=3u+3} 3u+3-u \geq 0 \Rightarrow u \geq -3 \xrightarrow{u < 1} -3 \leq u < 1 \text{ (II)} \end{cases}$$

$$D_g = ? \xrightarrow{(I) \cup (II)} \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \end{array} \rightarrow D_g = [-3, +\infty)$$

$$f(u) = \begin{cases} (a+1)(u+2) & ; u > 1 \\ 3a+2u & ; u \leq 1 \end{cases} \rightarrow f(a) = (a+1)(a+2) = 4a+2$$

$$f(-1) = 3a+2(-1) = 3a-2$$

$$2f(a) = f(-1) + a \Rightarrow 2(4a+2) = 3a-2 + a \Rightarrow 8a+4 = 4a-2 \Rightarrow 4a = -6 \Rightarrow a = -\frac{3}{2}$$

$$f(u) = \sqrt{u} + \frac{1}{\sqrt{u}} + 2 \Rightarrow \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \Rightarrow \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 \Rightarrow f(2-\sqrt{3}) \text{ (I)}$$

$$\frac{1}{\sqrt{2-\sqrt{3}}} \times \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{1}} = \sqrt{2+\sqrt{3}}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$$

$$\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 \Rightarrow \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2 \Rightarrow f(2+\sqrt{3}) \text{ (II)}$$

$$\frac{1}{\sqrt{2+\sqrt{3}}} \times \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}} = \frac{\sqrt{2-\sqrt{3}}}{\sqrt{1}} = \sqrt{2-\sqrt{3}}$$

$$\xrightarrow{(I)+(II)} \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 + \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2$$

$$= 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 \Rightarrow a \Rightarrow a^2 = 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} + 4 = 8 \Rightarrow a = \sqrt{8}$$

$$\Rightarrow 2(\sqrt{8}) + 4 = f(2-\sqrt{3}) + f(2+\sqrt{3})$$

$$\begin{cases} 2f(u) - 3f(-u) = 5u^2 - 2u \xrightarrow{x=u} 2f(u) - 3f(-u) = 5u^2 - 2u \\ 2f(-u) - 3f(u) = -5u^2 + 2u \xrightarrow{x=u} 2f(-u) - 3f(u) = -5u^2 + 2u \end{cases}$$

$$-5f(u) = 5u^2 + 2u \Rightarrow f(u) = \frac{5u^2 + 2u}{-5}$$

$$(u+2)f(u) - 3uf(u+2) = 5u^2 - 2(u+2) - 1$$

$$u = -2 \Rightarrow \begin{cases} 2f(0) = 5(-2)^2 - 2(-2) = 20 + 4 = 24 \xrightarrow{x=0} 2f(0) = 24 \\ 3f(0) = 5(-2)^2 + 2(-2) = 20 - 4 = 16 \xrightarrow{x=0} 3f(0) = 16 \end{cases}$$

$$2f(0) = 24 \Rightarrow f(0) = 12$$

$$f(u) = au + b$$

$$f\left(\frac{1}{u}\right) = \frac{a}{u} + b$$

$$\begin{cases} \textcircled{1} au + b + \frac{a}{u} + b = au + \frac{a}{u} + 2b = \frac{5u^2 - 2u + 2}{u} \Rightarrow au + \frac{a}{u} + 2b = 5u - 2 + \frac{2}{u} \\ \Rightarrow \begin{cases} a = 5 \\ b = 2 \end{cases} \Rightarrow f(-1) = 5(-1) + 2 = -3 \end{cases}$$