نگار صدائی

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$x\neq 0 \quad x\neq 1 \quad , \quad \frac{x-1}{x}-\frac{x}{x-1}\geq 0$

$\frac{(x-1)^2-x^2}{x(x-1)}\geq 0 \quad \frac{-2x+1}{x(x-1)}\geq 0$

x	$-\infty$		0		$\frac{1}{2}$		1		$+\infty$
$-2x+1$	$+$		$+$		0		$-$		$-$
$x(x-1)$	$+$		0	$-$	0	$+$	0	$-$	
	$-$		$\|$	$+$	0	$-$	$\|$	$+$	

$Df=(0,\frac{1}{2}]\cup(1,+\infty)$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{2}{x-1}+\frac{1}{x+2}}$

$x\neq 0 \quad x\neq -1 \quad x\neq 1 \quad x\neq -2 \quad x=-\frac{5}{4}$

$f(x)=\frac{\frac{-x+2}{(x+1)x}}{\frac{4x+5}{(x-1)(x+2)}}=\frac{(x-1)(x+2)(2-x)}{x(1+x)(5+4x)}$

$R-[-2,-\frac{5}{4},-1,0,1]$

(۲)

الف) $f(x)=\sqrt{((\frac{1}{3})^{-x}-9)(2^5-2^{2x})}$

$(3^{-x}-9)(2^5-2^{2x})\geq 0$

	$-\infty$		-2		$2,5$		$+\infty$
$(3^{-x}-9)$	$+$		0	$-$		$-$	
2^5-2^{2x}	$+$		$+$		0	$-$	
		$+$		$-$	0		$+$

$Df=(-\infty,-2]\cup[2,5,+\infty)$

۲ ب) $\sqrt[3]{x-1}+\sqrt{y+1}=2$

$\sqrt[3]{x-1}-2=\sqrt{y+1} \rightarrow y=\left[\sqrt[3]{(x-1)}-2\right]^2-1$

$Df\ [1,+\infty)$

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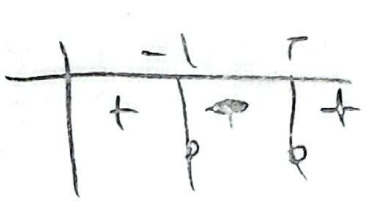
۳)

$f(x)=\frac{\log_{\frac{1}{2}}(x^2-x-2)}{\sqrt{x^2-1}+1}$

① $\sqrt{x^2-1} \rightarrow x^2-1\geq 0 \rightarrow -1\geq x\geq 1$

② $x^2-x-2>0 \rightarrow (x+1)(x-2)$



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۴)

$f(x)=\sqrt{c+ax-x^2}$ $\quad Df\ [-2,b]$

$-2,b$ ریشه‌ها $\rightarrow -2b=c \rightarrow b=+1/5$

x_1x_2

$x=-2 \rightarrow c-2a+4=0 \rightarrow a=-\frac{1}{2}$

$a+b=-\frac{1}{2}+\frac{3}{5}=-0.1 \rightarrow a+b=1$

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۵) $f(x)=\begin{cases} 2x-2 & x\geq 1 \\ x+2 & x<1 \end{cases}$

$g(x)=\sqrt{f(x)-x}$

$f(x)-x\geq 0 \rightarrow f(x)\geq x$ $\quad x\geq 1$

$2x-2-x\geq 0 \rightarrow x-2\geq 0 \rightarrow x\geq 2$

$x+2>x \rightarrow x+2>0 \rightarrow x>-2$ $\quad x<1$

$Dg\ [-2,1)\cup[2,+\infty)$

(۶)

$$f(x)=\begin{cases}(a+1)(x+2) & x\geq 1\\ 3a+4x & x<1\end{cases}$$

$$2f(0)=f(-2)+a$$

$$2\big(7(a+1)\big)=3a-4+a$$

$$14a+14=4a-4\rightarrow 10a=-18\rightarrow a=\frac{-18}{10}$$

(۷)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2=\frac{x+2\sqrt{x}+1}{\sqrt{x}}=\frac{(\sqrt{x}+1)^2}{\sqrt{x}+1-1}$$

$$f(x)=\frac{x^2}{x-1}\rightarrow f(2-\sqrt{3})=\frac{7-4\sqrt{3}}{1-\sqrt{3}}$$

$$f(2+\sqrt{3})=\frac{7+4\sqrt{3}}{1+\sqrt{3}}$$

$$\frac{7-4\sqrt{3}}{1-\sqrt{3}}+\frac{7+4\sqrt{3}}{1+\sqrt{3}}=\frac{7-4\sqrt{3}+7\sqrt{3}-12+7+4\sqrt{3}-7\sqrt{3}-12}{-2}$$

$$=\frac{-10}{-2}=5\longrightarrow$$ جواب (۵)

(۸)

$$3f(x)-2f(-x)=4x^2-x \qquad f(x)=?$$

$$2\,\big|\,-3f(x)+2f(-x)=4x^2+x$$

$$-5f(x)=20x^2+x\rightarrow f(x)=\frac{20x^2+x}{-5}$$

(۹)

$$x=0\rightarrow\quad 2f(0)-0=2m-1$$

$$x=-2\qquad 0+4f(0)=16+3m+2m-1=5m+15$$

$$9m-2=5m+15\rightarrow 4m=18\rightarrow m=\frac{18}{4}$$

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(1-) $f(x) = ax+b \rightarrow f(x) + f\left(\frac{1}{x}\right) = ax+b+\frac{a}{x}+b$

$$= \frac{ax^2+bx+a+bx}{x} = \frac{ax^2+2bx+a}{x}$$

$$10x^2 - 12x + c = ax^2 + 2bx + a$$

$a = 5$
$b = -6$ $\rightarrow f(x) = 5x - 6$

$f(-1) = -9$