مستی جبالی / یازدهم A / تکلیف ۴ :

۱) الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$: $\frac{(x-1)^2-x^2}{x(x-1)}=\frac{\cancel{x^2}-2x+1-\cancel{x^2}}{x^2-x}$: $\sqrt{\frac{1-2x}{x^2-x}}$ $\to \frac{1}{2}$; $\to 0,1$:

	0		$\frac{1}{2}$		1	
$+$		$-$		$+$		$-$

$\Rightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$: $\dfrac{\frac{x-2(x+1)}{x(x+1)}}{\frac{3(x+2)+x-1}{(x-1)(x+2)}}$: $\dfrac{\frac{-x-2}{x^2+x}}{\frac{4x+5}{x^2+x-2}}$: $\dfrac{-(x^2+x-2)(x+2)}{(x^2+x)(4x+5)}$

$x-2x-2=-x-2$; $1,-2$; -2

$3x+6+x-1=4x+5$; $-1,0$; $-\frac{5}{4}$

$\Rightarrow D_f=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

-2	$-\frac{5}{4}$	-1	0	1
$-$	$-$	$+$	$-$	$+$

(نیازی نیست ولی خب) $x=10$: $\frac{-(+)(+)}{(+)(+)}$

۲) الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)\left(32-4^x\right)}$ $\to \geq 0$

$(\frac{1}{3})^x-9=0$; $(\frac{1}{3})^x=9$: $x=-2$

$32-4^x=0$; $32=4^x$; $2^5=2^{2x} \Rightarrow 2x=5$; $x=\frac{5}{2}$

	-2		$\frac{5}{2}$	
$+$		$-$		$+$

D_f : $(-\infty,-2]\cup[2/5,+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$; $x\geq 1$ ، $y\geq -1$

$0\leq\sqrt[4]{x-1}\leq 3$: $0\leq x-1\leq 81$; $1\leq x\leq 82$

$0\leq\sqrt{y+1}\leq 3 \to 0\leq y+1\leq 9$; $-1\leq y\leq 8$

$\sqrt{y+1}=3-\sqrt[4]{x-1}$; $y=(3-\sqrt[4]{x-1})^2-1$

$D_f=[1,82]$

۳) $f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$ $\to x^2-x-2>0$ ، $x=-1,2$:

	-1		2	
$+$		$-$		$+$

: $(-\infty,-1)\cup(2,+\infty)$

$x^2-1\geq 0$; $-1,1$

	-1		1	
$+$		$-$		$+$

: $(-\infty,-1)\cup(1,+\infty)$

، $\sqrt{x^2-1}+1>0$; $\sqrt{x^2-1}\neq -1$ ✓

$\Rightarrow D_f=(-\infty,-1)\cup(2,+\infty)$

۴) $\sqrt{3+ax-x^2}$: $D_f=[-2,b]$ ، $a+b=?$

	-2		b	
$-$		$+$		$-$

$3-2a-4=0$

$-1-2a=0$

$a=-\frac{1}{2}$; $\sqrt{3-\frac{1}{2}x-x^2}$

$-b^2-\frac{1}{2}b+3=0$

$b^2+\frac{1}{2}b-3=0 \to b=-2,\frac{3}{2}$ ✓

$\times[-2,-2]\leftarrow\times$

$\Delta=\frac{1}{4}+12=\frac{49}{4}$: $b=\dfrac{-\frac{1}{2}\pm\frac{7}{2}}{2}$ $\to \frac{3}{2}$; $\to -2$

$\Rightarrow a+b=-\frac{1}{2}+\frac{3}{2}=1$

۵)

$$f(x)=\begin{cases}3x-2 & : x\geq 1 \to x\geq 1^{(+)} : 3x-2\geq x \to 2x\geq 2 \to x\geq 1 \checkmark\\ 2x+3 & : x<1 \to x<1^{(-)} : 2x+3\geq x \to \underline{x\geq -3}\end{cases}$$

$$g(x)=\sqrt{f(x)-x} \quad : ?D_f$$

$$f(x)-x\geq 0 \to f(x)\geq x \quad : \underline{D_f=[-3,+\infty)}$$

۶)

$$f(x)=\begin{cases}(a+1)(x+2) & : x>1\\ 4a+2x & : x\leq 1\end{cases} \quad , \; 2f(5)=f(-2)+a \quad , \underline{a=?}$$

$$\Rightarrow 2(\overbrace{(a+1)\times 7}^{7a+7})=(4a-4)+a$$

$$14a+14=4a-4 \to 10a=-18$$

$$\underline{a=-1.8}$$

۷)

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \quad \Rightarrow f(2-\sqrt{3})+f(2+\sqrt{3})=?$$

1.?

$$\Rightarrow \sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2 \quad \frac{(\sqrt{2-\sqrt{3}})\times(2+\sqrt{3})}{4-3} : \sqrt{2-\sqrt{3}}+\underbrace{(\sqrt{2-\sqrt{3}}\times(2+\sqrt{3}))}_{*_1}+2 \qquad ①$$

$$\Rightarrow \sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2 \quad \frac{(\sqrt{2+\sqrt{3}})\times(2-\sqrt{3})}{4-3} : \sqrt{2+\sqrt{3}}+\underbrace{(\sqrt{2+\sqrt{3}}\times(2-\sqrt{3}))}_{*_2}+2 \qquad ②$$

$$①+②=4+\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+*_1+*_2$$

۸)

$$\begin{cases}2f(x)-3f(-x)=4x^2-x & \xrightarrow{\times 2}\\ 2f(-x)-3f(x)=4x^2+x & \xrightarrow{\times 3}\end{cases} \quad \begin{cases}4f(x)-\cancel{6f(-x)}=8x^2-2x\\ \cancel{6f(-x)}-9f(x)=12x^2+3x\end{cases}$$

$$-5f(x)=20x^2+x \to \boxed{f(x)=-4x^2-\frac{1}{5}x}$$

۹)

$$f(\overset{0}{x})(x+2)-3f(\overset{-2}{x+2})x=4x^2-mx+3m-1$$

$$\xrightarrow{x=0} f(0)\times 2 \; \cancel{-3f(2)\times x}=-1+3m$$

$$\xrightarrow{x=-2} \cancel{f(-2)\times 0}+6f(0)=4\times 4+2m+3m-1 \quad : \begin{cases}2f(0)=3m-1\\ \underbrace{6f(0)}_{9m-3}=16+5m-1\end{cases} \Rightarrow 4m=18$$

$$\boxed{m=\frac{9}{2}}$$

$$\Rightarrow f(0)=\frac{-1+3m}{2} \; , \; m=\frac{9}{2} : \frac{-1+\frac{27}{2}}{2}=\frac{25}{4}$$

۱۰)

$$f(x)=ax+b \quad : \quad f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x} \quad : f(-1)=?$$

$$f(-1)+f(-1)=\frac{3+12+3}{-1}=-18 \longrightarrow f(-1)=-9$$