ماهک میرمقتدایی

۲۰

۱ ← اولی ←

$f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \rightarrow$

$$\frac{(x-1)^2-x^2}{x(x-1)} = \frac{1-2x}{x(x-1)} \geq 0$$

$(x-1)^2 \rightarrow x^2-2x+1$

$1-2x \rightarrow \frac{1}{2}$, $\quad x \rightarrow 0$, $\quad x-1 \rightarrow 1$

	0		$\frac{1}{2}$		1	
$+$	⊘	$-$	0	$+$	⊘	$-$

د

$$D_f: (-\infty,0)\cup[\tfrac{1}{2},1)$$

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۱ ← دومی ←

$$f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$$

$\rightarrow \frac{x-2x-1}{x(x+1)} = \frac{-(x+1)}{x(x+1)}$

$x \rightarrow 0 \neq$, $\quad x+1 \rightarrow -1 \neq$

$$\rightarrow \frac{3x+6+x-1}{(x-1)(x+2)} = \frac{4x+5}{(x-1)(x+2)}$$

$x-1 \rightarrow 1 \neq$, $\quad x+2 \rightarrow -2 \neq$

$$\frac{4x+5}{(x-1)(x+2)}=0 \rightarrow 4x+5=0 \rightarrow x=-\frac{5}{4} \neq$$

$$D_f = R-\{0,-1,1,-2,-\tfrac{5}{4}\}$$

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۲ ← اولی ←

$f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)} \rightarrow$

$$((\tfrac{1}{3})^x-9)(32-4^x) \geq 0$$

$(\frac{1}{3})^x-9 \rightarrow -2$, $\quad 32-4^x \rightarrow \frac{5}{2}$

$\rightarrow$

	-2		$\frac{5}{2}$	
$+$	0	$-$	0	$+$

د

$D_f = (-\infty, -۲] \cup [۵/۲, \infty)$

$\sqrt[۴]{x-1} + \sqrt{y+1} = ۳ \Rightarrow \sqrt{y+1} = ۳ - \sqrt[۴]{x-1}$ ⟸ بتوان ۲

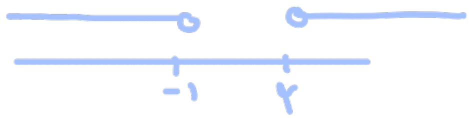
$y = ۹ - 1 + \sqrt{x-1} - ۶\sqrt[۴]{x-1} \Rightarrow y = ۸ - ۶\sqrt[۴]{x-1} + \sqrt{x-1}$

$\Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1$

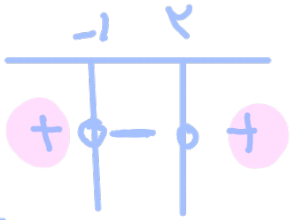
$۳ - \sqrt[۴]{x-1} \geq 0 \Rightarrow x - 1 \leq ۸۱ \Rightarrow x \leq ۸۲$ $\Big\} \Rightarrow D_f = [1, ۸۲]$

$f(x) = \dfrac{\log_۲ (x^۲ - x - ۲)}{\sqrt{x^۲ - 1} + 1}$

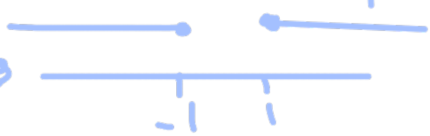
۹



① $\Rightarrow x^۲ - x - ۲ > 0 \Rightarrow (x-۲)(x+1) > 0$



② $\Rightarrow x^۲ - 1 \geq 0 \Rightarrow x^۲ \geq 1 \Rightarrow$



③ $\Rightarrow \sqrt{x^۲ - 1} + 1 \neq 0 \Rightarrow \sqrt{x^۲ - 1} \neq -1$ ✗ $\mathbb{R}$

همیشه + ⟸

① ∩ ② ∩ ③ $\Rightarrow D_f = (-\infty, -1) \cup (۲, +\infty)$

$\sqrt{۳ + ۶x - x^۲} \Rightarrow ۳ + ۶x - x^۲ \geq 0$ ۹ ⟸ ۲

$$x^2 - ax - 3 \leq 0$$

$$-2 \times b = 3 \Rightarrow b = \frac{3}{2} \Rightarrow a = -\frac{4}{2} + \frac{3}{2} = -\frac{1}{2} \Rightarrow$$

$$\Rightarrow a + b = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 1$$

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5 ←

$$f(x) = \begin{cases} 3x - 2, & x \geq 1 \\ 2x + 3, & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow$$

$$f(x) \geq x \rightarrow \left.\begin{array}{l} x \geq 1 \\ 2x + 3 \geq x \rightarrow x \geq -3 \end{array}\right\} \rightarrow D_f = [-3, +\infty)$$

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6 ←

$$f(x) = \begin{cases} (a+1)(x+2) & x \geq 1 \\ 3a + 2x & x < 1 \end{cases}$$

$$\left.\begin{array}{l} 2f(5) = f(-2) + a \\ \;\downarrow \qquad\quad \downarrow \\ 7(a+1) \quad 3a - 4 \end{array}\right\} \Rightarrow 14a + 14 = 4a - 4$$

$$10a = -18 \rightarrow a = -\frac{9}{5}$$

$$\frac{1}{2+\sqrt{3}} = 2-\sqrt{3} \Rightarrow f(\underbrace{2+\sqrt{3}}_{t}) + f\left(\frac{1}{2+\sqrt{3}}\right) \qquad \leftarrow ۷$$

$$f(t) + f\left(\frac{1}{t}\right) = 2\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right) + 4 \Rightarrow$$

$$2\left(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}}\right) + 4 \rightarrow$$

$$2\Big(\underbrace{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}_{S}\Big) + 4$$

ادامه سوال بعد از تمام سوالات

$$\left(2f(x) - 3f(-x) = 4x^2 - x\right) \times 3 \qquad \leftarrow ۸$$

$$\left(2f(-x) - 3f(x) = 4x^2 + x\right) \times 2$$

$$-5f(x) = 20x^2 + x = x(20x+1) \Rightarrow$$

$$f(x) = \frac{-x(20x+1)}{5}$$

(3)

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1 \qquad \leftarrow ۹$$

$x=0 \Rightarrow 2f(0) = (3m-1)\times -3 \quad 3m$

$x=-2 \Rightarrow 6f(0) = 16 + 3m - 1$

$0 = 18 - 4m \Rightarrow 18 = 4m \Rightarrow m = \frac{9}{2}$

$2f(0) = \frac{27}{2} - \frac{2}{2} = \frac{25}{2} \Rightarrow f(0) = \frac{25}{4} = 6{,}25$

$f(x) + f(\frac{1}{x}) = ax + b + \frac{a}{x} + b = \quad \leftarrow 10$

$a(x + \frac{1}{x}) + 2b = 3x - 12 + \frac{3}{x} \Rightarrow$

$\left.\begin{array}{l} a = 3 \\ b = -6 \end{array}\right\} \Rightarrow f(-1) = -1\times 3 - 6 = -9$

$S^2 = (2-\sqrt{3})(2+\sqrt{3}) + 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} \leftarrow$ رادیکال ۲

$\Rightarrow S^2 = 6$

$\sqrt{6}$

$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2S + 4 \rightarrow$

$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2\sqrt{6} + 4$