نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۵ کلاس ..... A

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$$\lim_{x\to 1}\frac{4x^2-\sqrt{x}+3}{5x^2-8x+3}\xrightarrow{\frac{0}{0}}\frac{(x-1)(4x-3)}{(5x-3)(x-1)}\Rightarrow\boxed{\frac{4x-3}{5x-3}}$$

$$\lim_{x\to 1}\frac{4x-3}{5x-3}=\boxed{\frac{1}{2}=0.5}$$

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$$\lim\frac{|\sqrt[3]{x}-1|-|\sqrt[3]{x+1}|}{x}\Rightarrow\begin{cases}0^+:\ \lim\frac{-\sqrt[3]{x}+1-\sqrt[3]{x}-1}{x}=\frac{-2x}{x}=\boxed{-2}\\ 0^-:\ \lim\frac{-\sqrt[3]{x}+1-\sqrt[3]{x}-1}{x}=\frac{-2x}{x}=\boxed{-2}\end{cases}$$

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$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}\xrightarrow{\frac{0}{0}}\frac{x-4}{\sqrt{x}-2}\times\frac{\sqrt{x}+2}{\sqrt{x}+2}=\frac{(x-4)(\sqrt{x}+2)}{(x-4)}=(\sqrt{x}+2)$$

ضرب در مزدوج مخرج

ضرب در مزدوج مخرج

$$\lim_{x\to 4}\sqrt{x}+2=2+2=\boxed{4}$$

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$$\lim_{x\to 3}\frac{x-\sqrt{2x}}{2x^2-x-6}\xrightarrow{\frac{0}{0}}\frac{x-\sqrt{2x}}{2x^2-x-6}\times\frac{x+\sqrt{2x}}{x+\sqrt{2x}}=\frac{x^2-2x}{(2x^2-x-6)(x+\sqrt{2x})}$$

ضرب در مزدوج صورت

ضرب در مزدوج صورت

$$\lim\frac{x^2-2x}{(2x^2-x-6)(x+\sqrt{2x})}=\frac{x(x-2)}{(2x+3)(x-2)(x+\sqrt{2x})}=\lim_{x\to 2}\frac{x}{(2x+3)(x+\sqrt{2x})}=\boxed{\frac{2}{28}=\frac{1}{14}}$$

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$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{a-x}}\xrightarrow{\frac{0}{0}=\text{H.O.P}}\text{مشتق از صورت و مخرج}\Rightarrow\frac{\frac{-1}{2\sqrt{x}}}{\frac{-(-1)}{2\sqrt{a-x}}}=\frac{-2\sqrt{a-x}}{2\sqrt{x}}=\frac{-\sqrt{a-x}}{\sqrt{x}}$$

$$\sqrt[n]{a^m}\Rightarrow\frac{ma'}{n\sqrt[n]{a^{n-m}}}$$

$$\lim_{x\to 1}\frac{-\sqrt{a-x}}{\sqrt{x}}=\frac{-2}{1}=\boxed{-2}$$

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$$\lim_{x\to 4}\frac{\sqrt{3x+4}-2}{\sqrt[3]{3x+7}-3}\xrightarrow{\frac{0}{0}}\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{3x+7}-3}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(3x+7)^2}+3\sqrt[3]{3x+7}+9}{\sqrt[3]{(3x+7)^2}+3\sqrt[3]{3x+7}+9}$$

$3x-12=3(x-4)$, $3x+7-27=3x-20=3(x-4)$

$$\lim_{x\to 4}\frac{3(x-4)}{3(x-4)}\times\frac{\sqrt[3]{(3x+7)^2}+3\sqrt[3]{3x+7}+9}{\sqrt{3x+4}+4}=\frac{3}{3}\times\frac{27}{8}=\boxed{\frac{81}{40}}$$

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$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\xrightarrow{\frac{0}{0}}\text{ضرب در مزدوج صورت و مخرج}\Rightarrow\lim_{x\to 1}\frac{3x+\sqrt{x}-4}{x-1}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2}$$

$$\Rightarrow\lim_{x\to 1}\frac{3x+\sqrt{x}-4}{x-1}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt{3x+\sqrt{x}}+2}\xrightarrow{\text{فاکتورگیری}}\frac{3}{4}\times\frac{(\sqrt{x}-1)(3\sqrt{x}+4)}{(\sqrt{x}+1)(\sqrt{x}-1)}=\frac{3}{4}\times\frac{7}{2}=\boxed{\frac{21}{8}}$$

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$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}\xrightarrow{\frac{0}{0}}\frac{(1+\cos x)(1-\cos x+\cos^2 x)}{1-\cos^2 x}\Rightarrow\frac{(1+\cos x)(1-\cos x+\cos^2 x)}{(1+\cos x)(1-\cos x)}$$

$$\lim_{x\to\pi}\frac{1-\cos x+\cos^2 x}{1-\cos x}=\frac{1-(-1)+1}{1-(-1)}=\boxed{\frac{3}{2}}$$

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$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\sin x-\cos x}{1}}=\frac{(\cos x-\sin x)}{-(\cos x)(\sin x-\cos x)}=\boxed{\frac{1}{-\cos x}}$$

$$\lim_{x\to\frac{\pi}{4}}-\frac{1}{\cos x}=\frac{-1}{\frac{\sqrt{2}}{2}}=\boxed{-\sqrt{2}}$$

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$$\lim_{x\to\frac{\pi}{4}}\frac{\tan^2 x-1}{\cos^2 x-\sin^2 x}\xrightarrow{\frac{0}{0}}\frac{\frac{\sin^2 x}{\cos^2 x}-\cos^2 x}{\frac{\cos^2 x-\sin^2 x}{1}}=\frac{(\sin^2 x-\cos^2 x)}{-(\cos^2 x)(\cos^2 x-\sin^2 x)}=\boxed{\frac{1}{-\cos^2 x}}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{1}{-\cos^2 x}=\frac{-1}{(\frac{\sqrt{2}}{2})^2}=-\frac{1}{\frac{2}{4}}=-\frac{1}{\frac{1}{2}}=\boxed{-2}$$

$\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$