نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

همه شیوه‌های زیر را بیست آورید.

۱

$$\lim_{x\to 1}\frac{4x^2-7x+3}{6x^2-12x+6}=\frac{0}{0}$$

$$\frac{4(x-1)(x-\frac{3}{4})}{6(x-1)(x-\frac{3}{6})}\Rightarrow\frac{4(x-\frac{3}{4})}{6(x-\frac{3}{6})}\xrightarrow{x\to 1}\frac{4(\frac{1}{4})}{6(\frac{1}{6})}=\frac{1}{2}$$

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$$\lim_{x\to 0}\frac{|4x-1|-|4x+1|}{x}=\frac{0}{0}$$

$$x\to 0^+\quad\frac{1-4x-4x-1}{x}=\frac{-8x}{x}=-8$$

$$x\to 0^-\quad\frac{1-4x-4x-1}{x}=\frac{-8x}{x}=-8$$

دارد

۳

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}$$

$$\frac{x-4}{\sqrt{x}-2}\times\frac{\sqrt{x}+2}{\sqrt{x}+2}\Rightarrow\frac{(x-4)(\sqrt{x}+2)}{(x-4)}=\sqrt{x}+2\xrightarrow{x\to 4}2+2=4$$

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$$\lim_{x\to 2}\frac{x-\sqrt{3x}}{x^2-x-1}\to\frac{0}{0}$$

$$\frac{x-\sqrt{3x}}{x^2-x-1}\Rightarrow\frac{x-\sqrt{3x}}{2(x-2)(x+\frac{1}{2})}\times\frac{x+\sqrt{3x}}{x+\sqrt{3x}}=$$

$$\Longrightarrow\frac{x^2-3x}{2(x-2)(x+\frac{1}{2})(x+\sqrt{3x})}\xrightarrow[\text{ساده}]{\text{بیشتر}}\frac{x(x-3)}{2(x-2)(x+\frac{1}{2})(x+\sqrt{3x})}\xrightarrow{x\to 2}\frac{1}{16}$$

$\frac{4}{2}+\frac{1}{2}=\frac{5}{2}$ ، $2+\sqrt{4}=4$

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$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{3-x}}=\frac{0}{0}$$

$$\frac{1-\sqrt{x}}{2-\sqrt{3-x}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{3-x}}{2+\sqrt{3-x}}=\frac{1-x}{4-(3-x)}\times\frac{2+\sqrt{3-x}}{1+\sqrt{x}}$$

$x-1$

$$\to\frac{-(x-1)}{(x-1)}\times\frac{2+\sqrt{3-x}}{1+\sqrt{x}}=-\frac{2+\sqrt{3-x}}{1+\sqrt{x}}\xrightarrow{x\to 1}-\frac{2+\sqrt{4}}{1+\sqrt{1}}=\frac{4}{2}=2$$

(۲) (۱)

Clinic

$\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{\sqrt[3]{3x+2}-2} = \frac{0}{0}$ $\quad * a^3-b^3=(a-b)(a^2+ab+b^2)$

$\frac{\sqrt{x^2+5}-3}{\sqrt[3]{3x+2}-2} \times \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3} \times \frac{(\sqrt[3]{3x+2})^2+2\sqrt[3]{3x+2}+4}{(\sqrt[3]{3x+2})^2+2\sqrt[3]{3x+2}+4} \Rightarrow \frac{x^2-4}{3x-6} \times \frac{(\sqrt[3]{3x+2})^2+2\sqrt[3]{3x+2}+4}{\sqrt{x^2+5}+3}$

$\frac{x^2-4}{3(x-2)} \times \frac{A}{\sqrt{x^2+5}+3} \xrightarrow{x\to 2} \frac{4}{3} \times \frac{4+4+4}{3+3} = \frac{12}{18}$ (A)

$\lim_{x\to 1} \frac{\sqrt{x^2+\sqrt{3x}}-2}{\sqrt[3]{3x}-1} = \frac{0}{0}$ $\quad * a^3-b^3=(a-b)(a^2+ab+b^2)$

$\frac{\sqrt{x^2+\sqrt{3x}}-2}{\sqrt[3]{3x}-1} \times \frac{\sqrt[3]{3x^2}+\sqrt[3]{3x}+1}{\sqrt[3]{3x^2}+\sqrt[3]{3x}+1} \times \frac{\sqrt{x^2+\sqrt{3x}}+2}{\sqrt{x^2+\sqrt{3x}}+2} = \frac{x^2+\sqrt{3x}-4}{3x-1} \times \frac{\sqrt[3]{3x^2}+\sqrt[3]{3x}+1}{\sqrt{x^2+\sqrt{3x}}+2}$

$\sqrt{3x}=t \Rightarrow \frac{t^2+t-6}{t^2-1} = \frac{(t-2)(t+3)}{(t-1)(t+1)} = \frac{(\sqrt{3x}+3)}{(\sqrt{3x}+1)} \times \frac{\sqrt[3]{3x^2}+\sqrt[3]{3x}+1}{\sqrt{x^2+\sqrt{3x}}+2} \xrightarrow{x\to 1} \frac{12}{8} \times \frac{1}{4} = \frac{12}{32}$

$\lim_{x\to \pi} \frac{1+\cos^3 x}{\sin^2 x} = \frac{0}{0}$ $\quad * (a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\frac{1+\cos^3 x}{\sin^2 x} = \frac{(1+\cos x)(1-\cos x+\cos^2 x)}{(1-\cos^2 x)} \xrightarrow{x\to\pi} \frac{(1+1+1)}{1+1} = \frac{3}{2}$

$(1-\cos)(1+\cos x)$

$\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{0}{0}$ $\quad \frac{1-\tan x}{\sin x-\cos x} = \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} \Rightarrow \frac{\cos x-\sin x}{\cos x} \Rightarrow \frac{\cos x-\sin x}{\sin x-\cos x} \searrow -(\cos x-\sin x)$

$\Rightarrow \frac{1}{\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}} \Rightarrow \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = -\sqrt{2}$

$\lim_{x\to \frac{3\pi}{4}} \frac{\tan^2 x-1}{\cos^2 x}$ $\quad \Rightarrow \frac{-(1-\tan x)(1+\tan x)}{\cos^2 x - \sin^2 x} \Rightarrow$

$-\frac{(\frac{\cos x-\sin x}{\cos x})(\frac{\cos x+\sin x}{\cos x})}{(\cos x-\sin x)(\cos x+\sin x)} \Rightarrow \frac{-1}{\cos^2 x} \Rightarrow \frac{-1}{\cos^2(\frac{3\pi}{4})} = \frac{-1}{(\frac{\sqrt{2}}{2})^2}$

$= -\frac{4}{2} = -2$