۱) $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3} = \frac{0}{0} = \frac{(x-1)(4x-3)}{(x-1)(5x-3)} = \frac{4x-3}{5x-3} = \frac{4-3}{5-3} = \frac{1}{2}$

۲) $\lim_{x\to 0} \frac{|3x-1|-|2x+1|}{x} = \frac{0}{0} = \frac{-3x+1-(-2x-1)}{x} = \frac{-x}{x} = -1$

$3x-1<0 \to x<\frac{1}{3}$ $\quad \frac{+}{-\frac{1}{3}+}$ ⇒ x در دو طرف منفی است

$2x+1<0 \to x<\frac{-1}{2}$ $\quad \frac{+}{-\frac{1}{2}+}$ ⇒ x در دو طرف مثبت است

۳) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \sqrt{4}+2 = 4$

۴) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} = \frac{0}{0} = \times \frac{x+\sqrt{2x}}{x+\sqrt{2x}} = \frac{x^2-2x}{(x+\sqrt{2x})(2x^2-x-6)} = \frac{x(x-2)}{(x+\sqrt{2x})(x-2)(2x+3)} = \frac{2}{(2+2)(8)} = \frac{1}{16}$

۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} = \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} = \frac{(1-x)(2+\sqrt{5-x})}{(4-(5-x))(1+\sqrt{x})} = \frac{(1-x)(2+\sqrt{5-x})}{-(1-x)(1+\sqrt{x})} = \frac{(2+2)}{-(2)} = -2$ $\quad \frac{0}{0}$

۶) $\lim_{x\to 4} \frac{\sqrt{2x+6}-4}{\sqrt[3]{3x+7}-3} = \times \frac{\sqrt{2x+6}+4}{\sqrt{2x+6}+4} \times \frac{\sqrt[3]{(3x+7)^2}+3\sqrt[3]{3x+7}+9}{\sqrt[3]{(3x+7)^2}+3\sqrt[3]{3x+7}+9} = \frac{(2x-8)(\sqrt[3]{(3x+7)^2}+3\sqrt[3]{3x+7}+9)}{(3x-12)(\sqrt{2x+6}+4)} = \frac{2(x-4)(9+9+9)}{3(x-4)(4+4)} = \frac{54}{24} = \frac{9}{4}$ $\quad \frac{0}{0}$

۷) $\lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} = \times \frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2} \times \frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1} = \frac{(\sqrt[3]{x}-1)(3\sqrt{x}+4)}{(3x+\sqrt{x}-4)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} = \frac{(\sqrt[3]{x}-1)(3\sqrt{x}+4)}{(3x+4\sqrt{x}-4)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} = \frac{(\sqrt[3]{x}-1)(3\sqrt{x}+4)}{(\sqrt{x}+2)(3\sqrt{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} = \frac{(1+1)(3+4)}{(4+2)(1+1+1)} = \frac{14}{18} = \frac{7}{9}$ $\quad \frac{0}{0}$

۸) $\lim_{x\to \pi} \frac{1+\cos^2 x}{\sin^2 x} = \frac{(1+\cos x)(1+\cos^2 x-\cos x)}{1-\cos^2 x} = \frac{1+1+1}{1+1} = \frac{3}{2}$ $\quad \frac{0}{0}$

$(1-\cos x)(1+\cos x)$

۹) $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} = \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \frac{\frac{\cos x-\sin x}{\cos x}}{-(\cos x-\sin x)} = \frac{-1}{\cos x} = \frac{-1}{\cos\frac{\pi}{4}} = \frac{-1}{\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$ $\quad \frac{0}{0}$

۱۰) $\lim_{x\to \frac{\pi}{4}} \frac{\tan^2 x-1}{\cos 2x} = \frac{\frac{\sin^2 x}{\cos^2 x}-1}{\cos^2 x-\sin^2 x} = \frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{-(\cos^2 x-\sin^2 x)} = \frac{-1}{\cos^2 x} = \frac{-1}{(\frac{\sqrt{2}}{2})^2} = -2$ $\quad \frac{0}{0}$

$\frac{1}{2}$