

Limit Algebra Properties

$$\lim_{x \rightarrow 1} \frac{r^x - 1}{\omega x^r - 1} = \frac{0}{0} \text{ form} = \lim_{x \rightarrow 1} \frac{(x-1)(rx-r)}{(x-1)(\omega x-r)} \quad (1)$$

$$= \lim_{x \rightarrow 1} \frac{rx-r}{\omega x-r} = \frac{r}{r}$$

$$\lim_{x \rightarrow 0} \frac{|rx-1| - |rx+1|}{x} = \frac{0}{0} \text{ form} \quad (2)$$

$$\begin{matrix} \xrightarrow{0^+} \\ \xrightarrow{0^-} \end{matrix} \frac{|0^+-1| - |0^++1|}{x} = \frac{-rx+1 - rx-1}{x} = -2$$

$$\frac{|0^- -1| - |0^- +1|}{x} = \frac{-rx+1 - rx-1}{x} = -2$$

$$\lim_{x \rightarrow \epsilon} \frac{x-\epsilon}{\sqrt{x}-r} = \frac{0}{0} \text{ form} \Rightarrow \frac{(\sqrt{x}-r)(\sqrt{x}+r)}{\sqrt{x}-r} = \lim_{x \rightarrow \epsilon} \sqrt{x}+r = \epsilon \quad (3)$$

$$\lim_{x \rightarrow r} \frac{x-\sqrt{rx}}{rx^r - x - r} = \frac{0}{0} \text{ form} \Rightarrow \frac{x-\sqrt{rx}}{(x-r)(x+\frac{r}{x})} \times \frac{\infty \cdot r}{\infty \cdot r} = \frac{x^r - rx}{(x-r)(x+\frac{r}{x})(\epsilon)}$$

$$= \frac{r}{(r+\frac{r}{r})\epsilon} = \frac{r}{2\epsilon} = \frac{1}{2}$$

$$\lim_{x \rightarrow 1} \frac{1-\sqrt{x}}{r-\sqrt{a-x}} \times \frac{r \cdot r}{r \cdot r} \times \frac{\infty \cdot r}{\infty \cdot r} = \frac{(1-x)\epsilon}{(r-\omega+x)r} = \frac{(1-x)\epsilon}{(-1+x)r} = \frac{\epsilon - \epsilon x}{-r+rx} \quad (4)$$

$$= \frac{r(1-x)}{-r+rx} = -1$$

$$\lim_{x \rightarrow \epsilon} \frac{\sqrt{rx+\epsilon} - \epsilon}{\sqrt{\omega x+r} - r} = \frac{0}{0} \text{ form} \times \frac{\infty \cdot r}{\infty \cdot r} \times \frac{r \cdot \epsilon}{r \cdot \epsilon} = \frac{(rx+\epsilon-1\epsilon)r}{(\omega x+r-r)r} \quad (5)$$

$$= \frac{(rx-1r)(r\epsilon)}{(\omega x-r)r} = \frac{(x-\epsilon) \times r \times r\epsilon}{(x-\epsilon) \times \omega \times r} = \frac{r}{\omega}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{3x} + \sqrt{x} - 2}{\sqrt{x} - 1} = \frac{0}{0} \times \frac{?}{?} = \frac{(3x + \sqrt{x} - \varepsilon) \times \tau}{(x-1) \times \varepsilon} \quad (9)$$

$$= \frac{3x + \sqrt{x} - 2}{\varepsilon x - \varepsilon} = \frac{3(\sqrt{x} - 1)(\sqrt{x} + \frac{\varepsilon}{\tau})}{\varepsilon(\sqrt{x} - 1)(\sqrt{x} + 1)} = \frac{3}{1}$$

$$\lim_{x \rightarrow \pi} \frac{1 + \cos^2 x}{\sin^2 x} = \frac{(1 + \cos x)(1 + \cos^2 x - \cos x)}{(1 - \cos x)(1 + \cos x)} = \frac{1 + 1 - (-1)}{2} = \frac{3}{2} \quad (10)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \tan x}{\sin x - \cos x} = \frac{\cos x - \sin x}{\cos x} = -\frac{1}{\cos x} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2} \quad (11)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^2 x - 1}{\cos^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \frac{1}{\frac{1}{2}} = 2 \quad (12)$$