پرنیا دیباجی — یازدهم تکمیلی‌ها — گروه A — تکلیف ۳

۱)

الف) $y = \dfrac{x+3}{2x^3+3x^2-8x+3} \Rightarrow 2x^3+3x^2-8x+3 \neq 0$

$$\begin{array}{l|l} 2x^3+3x^2-8x+3 & x-1 \\ \cline{2-2} -2x^3+2x^2 & 2x^2+5x-3 \\ \hline 5x^2-8x+3 & \\ -5x^2+5x & \\ \hline -3x+3 & \\ +3x-3 & \\ \hline 0 & \end{array}$$

$2x^2+5x-3 \Rightarrow (2x-1)(x+3)(x-1) \neq 0$

$\Rightarrow \boxed{x \neq \frac{1}{2}, -3, 1} \Rightarrow \boxed{D_f = \mathbb{R} - \{-3, \frac{1}{2}, 1\}}$

ب) $y = \dfrac{x+3}{2x^3+9x^2+10x+3} \Rightarrow 2x^3+9x^2+10x+3 \neq 0$

$$\begin{array}{l|l} 2x^3+9x^2+10x+3 & x+1 \\ \cline{2-2} -2x^3-2x^2 & 2x^2+7x+3 \\ \hline 7x^2+10x+3 & \\ -7x^2-7x & \\ \hline 3x+3 & \\ -3x-3 & \\ \hline 0 & \end{array}$$

$2x^2+7x+3 \Rightarrow (2x+1)(x+3)(x+1) \neq 0$

$\Rightarrow x \neq \frac{-1}{2}, -3, -1$

$\Rightarrow \boxed{D_f = \mathbb{R} - \{-3, -1, \frac{-1}{2}\}}$

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۲)

الف) $y = \dfrac{x+3}{x^3-2x^2+2x-1} \Rightarrow x^3-2x^2+2x-1 \neq 0 \Rightarrow$

$$\begin{array}{l|l} x^3-2x^2+2x-1 & x-1 \\ \cline{2-2} -x^3+x^2 & x^2-x+1 \\ \hline -x^2+2x-1 & \\ +x^2-x & \\ \hline x-1 & \\ -x+1 & \\ \hline 0 & \end{array}$$

$\Downarrow$

$(x-1)\underbrace{(x^2-x+1)}_{\text{همیشه مثبت}}$

$\Rightarrow x-1 \neq 0 \Rightarrow x \neq 1 \Rightarrow \boxed{D_f = \mathbb{R} - \{1\}}$

ب) $y = \sqrt{\dfrac{x+3}{x^3-2x^2+2x-1}} \Rightarrow \dfrac{x+3}{x^3-2x^2+2x-1} \geq 0 \Rightarrow x^3-2x^2+2x-1 > 0 \Rightarrow (x-1)\underbrace{(x^2-x+1)}_{\text{همیشه مثبت}} > 0$

$\Rightarrow x-1 > 0 \Rightarrow x > 1$ و $x+3 \geq 0 \Rightarrow x \geq -3 \Rightarrow$

	-3		1	
$+$		$-$		$+$

$\Rightarrow$

$\boxed{D_f = (-\infty, -3] \cup (1, +\infty)}$

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۳)

$y = \dfrac{2}{x^2-5|x-1|-2x+5} \Rightarrow x^2-5|x-1|-2x+5 \neq 0$

$$\begin{cases} \text{if } x \geq 1 \Rightarrow |x-1| = x-1 \Rightarrow x^2-5(x-1)-2x+5 \neq 0 \Rightarrow x^2-7x+10 \neq 0 \Rightarrow (x-5)(x-2) \neq 0 \\ \quad \Rightarrow x \neq 5, x \neq 2 \\ \text{if } x < 1 \Rightarrow |x-1| = -x+1 \Rightarrow x^2-5(-x+1)-2x+5 \neq 0 \Rightarrow x^2+3x \neq 0 \Rightarrow x(x+3) \neq 0 \\ \quad \Rightarrow x \neq -3, x \neq 0 \end{cases}$$

$\Rightarrow D_f = \mathbb{R} - \{-3, 0, 2, 5\}$

۴)

$$y=\frac{x+3}{|2x+1|-|x+3|} \Rightarrow |2x+1|-|x+3| \neq 0 \Rightarrow |2x+1| \neq |x+3| \xRightarrow{\text{توان ۲}} 4x^2+4x+1 \neq x^2+6x+9$$

$$\Rightarrow 3x^2-2x-8 \neq 0 \Rightarrow \begin{cases} \Delta = b^2-4ac = 4+96=100 \\ x=\dfrac{-b\pm\sqrt{\Delta}}{2a} = \dfrac{+2\pm 10}{6} \Rightarrow x\neq 2,\ x\neq -\dfrac{4}{3} \end{cases} \Rightarrow \boxed{D_f = \mathbb{R} - \left\{-\frac{4}{3}, 2\right\}}$$

$$y=\sqrt{|2x+1|-|x+3|} \Rightarrow |2x+1|-|x+3| \geq 0 \Rightarrow |2x+1| \geq |x+3| \Rightarrow 3x^2-2x-8 \geq 0$$

$$\Rightarrow (x-2)(3x+4) \geq 0 \Rightarrow \begin{cases} x=2 \\ x=-\dfrac{4}{3} \end{cases}$$

x		$-\frac{4}{3}$		2	
	$+$	0	$-$	0	$+$

$$\Rightarrow \boxed{D_f = \left(-\infty, -\frac{4}{3}\right] \cup [2, +\infty)}$$

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۵)

الف) $y=\log_2\left(1-\log_3 x\right) \Rightarrow \begin{cases} 1-\log_3 x > 0 \Rightarrow \log_3 x < 1 \Rightarrow x<3 \\ x>0 \end{cases} \Rightarrow D_f=(0,3)$

ب) $y=\log_2\left(1-\log_{\frac{1}{2}} x\right) \Rightarrow \begin{cases} 1-\log_{\frac{1}{2}} x > 0 \Rightarrow \log_{\frac{1}{2}} x < 1 \Rightarrow x > \frac{1}{2} \\ x>0 \end{cases} \Rightarrow \boxed{D_f=\left(\frac{1}{2}, +\infty\right)}$

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۶)

$$f(x)=\sqrt{\log_{0/5} \log_5 (2x-1)}$$

$$\Rightarrow \begin{cases} \log_{0/5} \log_5 (2x-1) \geq 0 \Rightarrow \log_5(2x-1) \leq 1 \Rightarrow 2x-1 \leq 5 \Rightarrow x \leq 3 \\ \log_5(2x-1) > 0 \Rightarrow x>1 \\ 2x-1>0 \Rightarrow x > \frac{1}{2} \end{cases} \Rightarrow \boxed{D_f=(1,3]}$$

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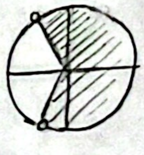
۷)

الف) $y=\log(2\cos x+1) \Rightarrow 2\cos x+1>0 \Rightarrow 2\cos > -1 \Rightarrow \cos > -\frac{1}{2}$



$$\Rightarrow D_f=\mathbb{R}-\left[\frac{2\pi}{3}+2k\pi,\ \frac{4\pi}{3}+2k\pi\right]$$

ب) $y=\sqrt{\log\frac{x-1}{x+1}} \Rightarrow \begin{cases} \dfrac{x-1}{x+1}>0 \Rightarrow \begin{cases} x>1 \\ x<-1 \end{cases} \Rightarrow \begin{cases} \text{if } x>1 \Rightarrow \dfrac{x-1}{x+1} \geq 1 \Rightarrow x-1 \geq x+1 \Rightarrow -1 \geq 1 \text{ غقق} \\ \text{if } x<-1 \Rightarrow \dfrac{x-1}{x+1} \geq 1 \Rightarrow x-1 \leq x+1 \Rightarrow -1 \leq 1 \ ✓ \end{cases} \\ \log\left(\dfrac{x-1}{x+1}\right) \geq 0 \Rightarrow \dfrac{x-1}{x+1} \geq 1 \end{cases}$

$$\Rightarrow \boxed{D_f=(-\infty, -1)}$$

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۸)

$f(x)=\sqrt{(a+2)x^2+ax+b} \Rightarrow a+2=0 \Rightarrow a=-2$

$D_f=(-\infty, 3]$ $\quad b=?$ $\quad \Rightarrow (a+2)x^2+ax+b \geq 0 \Rightarrow -2x+b=0 \xRightarrow{x=3} \boxed{b=6}$

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۹)

$f(x)=\sqrt{x^2+2x+2-m^2}$

$D_f=\mathbb{R} \Rightarrow \Delta \leq 0 \Rightarrow 4-(4\times 1(2-m^2)) \leq 0 \Rightarrow 4-8+4m^2 \leq 0 \Rightarrow 4m^2-4 \leq 0 \Rightarrow 4(m^2-1) \leq 0 \Rightarrow m^2-1 \leq 0$

$\Rightarrow m^2 \leq 1 \Rightarrow -1 \leq m \leq 1 \Rightarrow \boxed{1-(-1)=2}$

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۱۰)

$$f(x)=\frac{\sqrt{4-x^2}}{[x]+[-x]+1} \Rightarrow \begin{cases} 4-x^2 \geq 0 \Rightarrow -2 \leq x \leq 2 \\ [x]+[-x] \neq -1 \Rightarrow \begin{cases} 0 & x\in\mathbb{Z} \\ -1 & x\notin\mathbb{Z} \end{cases} \Rightarrow D_f=\mathbb{Z} \end{cases} \xRightarrow{\cap} D_f=[-2,2]$$

$\Rightarrow$ ۵ عدد صحیح در دامنه‌ی تابع قرار دارند