

المسألة: A function is given

$$f(x) = \frac{x+1}{x^2+x^2-1x+1} \xrightarrow{*} \frac{x^2+x^2-1x+1}{x^2+x^2-1x+1} \cdot \frac{x-1}{x^2+x^2-1x+1} = \frac{x-1}{(x-1)(x+1)}$$

* $\frac{x^2+x^2-1x+1}{x^2+x^2-1x+1} = 1$

$$\rightarrow \frac{x+1}{(x-1)(x+1)(x-1)} = \frac{1}{(x-1)^2}$$

$$D_f = \mathbb{R} - \{-1, 1\}$$

$$f(x) = \frac{x+1}{x^2+9x^2+1 \cdot x+1} \xrightarrow{*} \frac{x^2+9x^2+1 \cdot x+1}{x^2+9x^2+1 \cdot x+1} \cdot \frac{x+1}{x^2+9x^2+1 \cdot x+1} = \frac{x+1}{(x+1)(x^2+9x^2+1 \cdot x+1)}$$

* $\frac{x^2+9x^2+1 \cdot x+1}{x^2+9x^2+1 \cdot x+1} = 1$

$$\rightarrow \frac{x+1}{(x+1)(x^2+9x^2+1 \cdot x+1)} = \frac{1}{x^2+9x^2+1 \cdot x+1}$$

$$D_f = \mathbb{R} - \{-1, -1/2, -1\}$$

$$f(x) = \frac{x+1}{x^2-x^2+1x-1} \xrightarrow{f(x)} \frac{x^2-x^2+1x-1}{x^2-x^2+1x-1} \cdot \frac{x-1}{x^2-x^2+1x-1} = \frac{x-1}{(x-1)(x^2-x^2+1x-1)}$$

$$\rightarrow \frac{x+1}{(x^2-x^2+1x-1)(x-1)} + \frac{1}{x-1} = \frac{x+1}{(x-1)^2} + \frac{1}{x-1}$$

$$D_f = \mathbb{R} - \{1\}$$

$$\frac{x-1}{x-1} = 1$$

$$b) y = \sqrt{\frac{x^k}{x^k - 2n^k + n^k - 1}} \rightarrow \frac{x^k}{(x-1)(x^k - x + 1)} \quad \text{--- } k$$

$$+ \frac{-k}{x-1} + \frac{1}{x^k - x + 1}$$

$$D_f = (1, +\infty) \cup (-\infty, -k]$$

ii) $y = \log_r (1 - \log_r^n)$

①: $\log_r^n \rightarrow n \gamma$

$$\frac{\overline{\overline{\overline{0}}}}{0 \quad 1 \quad k} \rightarrow D_f = (0, k)$$

②: $\log_r^n < 1 \rightarrow n < k$

iii) $y = \log_r (1 - \log_{1/r}^n)$

①: $\log_{1/r}^n \rightarrow n \gamma$

$$\frac{\overline{\overline{\overline{0}}}}{0 \quad 1 \quad 1/r} \rightarrow D_f = (1/r, +\infty)$$

②: $\log_{1/r}^n < 1 \rightarrow n < 1/r$

$$y = \frac{x^k}{x^k - a|x-1| - 2x + a} \quad \text{--- } k$$

$$x^k - a|x-1| - 2x + a \neq 0$$

$$+ \frac{x^k}{x^k - a|x-1| - 2x + a} = (x-a)(x-1) \rightarrow x \neq 1, a$$

$$- \frac{x^k}{x^k + k} = x(x+k) + n \neq k, 0 \rightarrow D_f = \mathbb{R} \setminus \{a, k, 0\}$$

ii) $\frac{x^k}{|2x+1| + |x+k|} \rightarrow \frac{|2x+1| + |x+k|}{\sum x^k, k, 1 \neq x^k, 4x+9} \quad \text{--- } k$

$$x^k - 2x - 1 \neq 0 \rightarrow x^k - 2x - 1 \neq 0 \rightarrow (x-1)(x+2) \neq 0 \rightarrow D_f = \mathbb{R} \setminus \{-4, 1, 2\}$$

$$b) \sqrt{|2x+1| - |x+2|} \rightarrow |2x+1| \geq |x+2|$$

$$x^2 - x - 1 \geq 0 \rightarrow \frac{-(-1) \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$D_f = (-\infty, -\frac{1+\sqrt{5}}{2}] \cup [\frac{1+\sqrt{5}}{2}, +\infty)$$

$$c) \sqrt{\log_{1/r} (x-1)}$$

①: $x-1 \geq 0 \rightarrow x \geq 1$

②: $\log_{1/r} (x-1) \geq 0 \rightarrow x-1 \geq 1 \rightarrow x \geq 2$

③: $\log_{1/r} (x-1) \leq 1 \rightarrow x-1 \leq r \rightarrow x \leq r+1$

$$D_f = (1, r+1]$$

$$d) y = \log(r \cos x + 1) \rightarrow r \cos y - 1 \rightarrow \cos y \geq \frac{1-r}{r}$$

$$D_f = (r \cos \alpha - 1, r \cos \alpha + 1)$$

$$e) \sqrt{\log \frac{x-1}{x+1}}$$

①: $\log \frac{x-1}{x+1} \geq 0 \rightarrow \frac{x-1}{x+1} \geq 1 \rightarrow x-1 \geq x+1 \rightarrow -2 \geq 0$

$$\frac{x-1-x-1}{x+1} = \frac{-2}{x+1} \geq 0 \rightarrow x+1 \leq 0 \rightarrow x \leq -1$$

$$D_f = (-\infty, -1]$$

②: $\frac{x-1}{x+1} \leq 1 \rightarrow x-1 \leq x+1 \rightarrow -2 \leq 0$

$$D_f = (-\infty, -1] \cup (1, +\infty)$$

$$f(x) = \sqrt{(a+x)x^2 + ax + b}$$

as $f(x)$ is real $f(x) \geq 0$

$$\rightarrow -x^2 + b \geq 0 \rightarrow b \geq x^2$$

$$D_F = \mathbb{R} \setminus \{0\}$$

$$f(m) = \sqrt{m^2 + 2m + 2 - m^2} \rightarrow \Delta = 0 \quad \Sigma - (\Sigma + 1 + 2 - m^2) = 0 \quad -9$$

$$\Sigma - 1 + \Sigma m^2 = 0 \rightarrow \Sigma m^2 = 2 \rightarrow m = \pm 1 \rightarrow 1 - (-1) = 2$$

$$f(m) = \frac{\sqrt{2 - m^2}}{[m] + [-m] + 1} \quad \Sigma - m^2 \neq 0 \rightarrow m \neq \pm 1 \rightarrow -1 \leq m \leq 1 \quad (1.)$$

$$[m] + [-m] + 1 \sim [m] + [-m] \neq -1 \rightarrow \mathbb{Z}$$

$$\xrightarrow{D_F} \{ -2, -1, 0, 1, 2 \} \rightarrow \text{max, min}$$