سایا هادیان

شماره ۲۸

۱۸,۷۵

① $\frac{1}{\sqrt{\cos^2 x}} - \frac{1}{\cos x} = \frac{1-\sin x}{|\cos x|} \Rightarrow \frac{1}{|\cos x|} - \frac{\sin x}{\cos x} = \frac{1-\sin x}{|\cos x|} \Rightarrow \frac{\cos x}{\cos x} = \frac{\sin x}{|\cos x|}$

$\Rightarrow \cos x = |\cos x|$ ⑤

$\cot x = \frac{\cos x}{\sqrt{1-\cos^2 x}} \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos x}{|\sin x|} \Rightarrow \sin x = |\sin x|$ ②

$\Rightarrow$ $\cos x$ و $\sin x$ هم علامت

$\boxed{\text{ناحیه اول}}$

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② $-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x \le 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \le 1$

$\sin 2x = \frac{m-1}{4}$

$\Rightarrow \boxed{-1 < m \le 5}$ ⑤

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③ $\tan x + \cot x = \frac{2}{\sin 2x} = -3 \Rightarrow \sin 2x = -\frac{2}{3} \rightsquigarrow \sin 2x = 2\sin x\cos x = -\frac{2}{3} \rightarrow \sin x\cos x = -\frac{1}{3}$

$\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi$ $\quad \sin x + \cos x = \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x\cos x)} = \frac{1}{\sqrt{\frac{1}{3}} \times 1\frac{1}{3}} = \boxed{\frac{3\sqrt{3}}{4}}$

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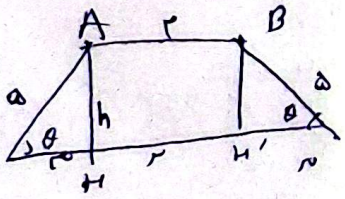
④



$\cos\theta = \frac{DH}{AD} = \frac{CH'}{BC} = \frac{h}{10} \Rightarrow DH = CH' = 3$

$AB = HH' = 2$

$\sin\theta = \sqrt{1-0.36} = 0.8 = \frac{h}{5} \Rightarrow h = 4$ ⑤

$S = \frac{1}{2} \times 4 \times (2+8) = \boxed{20}$

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⑤ $\tan(285)\tan(-105) - \sin(1095)\cos(255) = \tan(\frac{3\pi}{2}+15)\tan(\pi+15) - \sin(6\pi+15)\cos(\frac{3\pi}{2}-15)$

$\Rightarrow -\cot 15 \tan 15 - \sin 15(-\sin 15) = \sin^2 15 - 1 = -\cos^2 15 = k\cos^2 15 \Rightarrow \boxed{k = -1}$ ⑤

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⑥ $\sqrt{3}\cos 210 \sin 240 - \sqrt{2}\sin 135 \cos 150 = \sqrt{48} - \frac{\sqrt{3}}{2}\sin(\frac{3\pi}{2} - 27) - \sqrt{2} \times \frac{\sqrt{2}}{2}\cos(\pi - 27)$

$= +\frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27 = \boxed{\frac{15}{2}}$ ⑤

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$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{\pi}{6}\right)\frac{2\sin\left(4\times\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{12}\right)} = 16\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\sin\frac{4\pi}{3}$$

$$\frac{\sin\left(\frac{4\pi}{3}\right)}{\frac{1-\cos\frac{\pi}{6}}{2}} = \frac{2\left(-\frac{\sqrt{3}}{2}\right)^2}{1-\cos\frac{\pi}{6}} = \frac{\frac{3}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{3}{2-\sqrt{3}} = \boxed{6-3\sqrt{3}}$$

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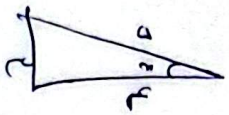
$$\frac{1-\sin x}{1+\sin x} = 4 \Rightarrow 1-\sin x = 4+4\sin x \Rightarrow \sin x = -\frac{3}{5}$$



$\pi < x < 2\pi$

$\Rightarrow \cos x = -\frac{4}{5}$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{5}}{1-\frac{4}{5}} = \boxed{-3}$$

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$$\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2} \longrightarrow \frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\tan\frac{\theta}{2}} + \frac{1}{\tan\frac{\theta}{2}} = 2\cot\frac{\theta}{2} \Rightarrow \boxed{\text{گزینه ۲}}$$

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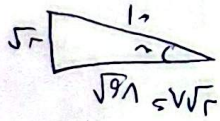
$\sin\alpha = \frac{\sqrt{2}}{10}$



$\pi < \alpha < \frac{3\pi}{2}$

$\cos\alpha = -\frac{7\sqrt{2}}{10}$

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\left(\frac{3\pi}{4}+\alpha\right) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha = \left(-\frac{\sqrt{2}}{2}\times-\frac{7\sqrt{2}}{10}\right)-\left(\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}\right) = \boxed{0/6}$$

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۷) $f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$

$$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{2+\sqrt{3}}{4}$$

$$16\left(\frac{2+\sqrt{3}}{4}\right)\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{3(2+\sqrt{3})}{16}$$