سوال دوم.

$\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$ $\qquad 1-\cos^2\alpha = \sin^2\alpha$ ۱۱

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} =$$

$$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha = |\cos\alpha| \Rightarrow \cos\alpha > 0$$

$$\cot\alpha = -\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = -\frac{\cos\alpha}{\sqrt{\sin^2\alpha}} \Rightarrow$$

$$\frac{1}{\sin\alpha} = \frac{1}{|\sin\alpha|} \Rightarrow |\sin\alpha| = -\sin\alpha \Rightarrow \sin\alpha < 0$$

پس α در ناحیه ۴ مثلثاتی است!

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \xrightarrow{\times 2} -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$ (۲)

$$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \rightarrow -\frac{1}{2} < \sin 2x = \frac{m-1}{4} \leq 1$$

$$\xrightarrow{\times 4} -2 < m-1 \leq 4 \rightarrow -1 < m \leq 5 \rightarrow m \in [-1, 5]$$

(۷)

$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{2\pi}{24}\right)\cos^2\left(\frac{4\pi}{24}\right)\cos^2\left(\frac{8\pi}{24}\right)\cos^2\left(\frac{16\pi}{24}\right)$$

$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$$

$$\cos^2\left(\frac{\pi}{12}\right) = \frac{1}{2}\left(1+\cos\left(\frac{2\pi}{12}\right)\right) = \frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right) = \frac{2+\sqrt{3}}{4}$$

$$\cos^2\left(\frac{2\pi}{3}\right) = \cos^2\left(\pi - \frac{\pi}{3}\right) = \left(-\cos\frac{\pi}{3}\right)^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$f\left(\frac{\pi}{24}\right) = 16\left(\frac{2+\sqrt{3}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right)^2 \times \frac{1}{4} \Rightarrow$$

$$f\left(\frac{\pi}{24}\right) = \frac{6+3\sqrt{3}}{16}$$

۸-

$$1-\sin x = 4+4\sin x \Rightarrow 5\sin x = -3$$

$$\sin x = -\frac{3}{5}$$

$$\cos x = \pm\sqrt{1-\sin^2 x} = \pm\sqrt{1-\frac{9}{25}} = \pm\frac{4}{5}$$ (انتهای کمان در ربع سوم →)

$$\cos x = -\frac{4}{5}$$

$$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2} = \frac{1-\frac{4}{5}}{2} = \frac{1}{10}$$

$$\tan^2\left(\frac{x}{2}\right)+1 = \frac{1}{\underbrace{\cos^2\left(\frac{x}{2}\right)}_{\frac{1}{10}}} = 10 \qquad \tan^2\left(\frac{x}{2}\right) = 9$$

$$\tan\left(\frac{x}{2}\right) = \pm 3$$

$\frac{x}{2} = \frac{\pi}{2} + \frac{\alpha}{2}$ و $0 < \frac{\alpha}{2} < \frac{\pi}{4}$ ⇒ انتهای $\frac{x}{2}$ در ربع دوم ⇒ $\tan\left(\frac{x}{2}\right) < 0$

(19)

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{\sin^2\theta+(1-\cos^2\theta)}{(1-\cos\theta)\sin\theta}=\frac{2\sin^2\theta}{(1-\cos\theta)\sin\theta}$$

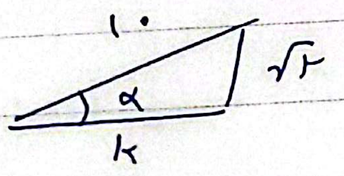
$$=\frac{2\sin\theta}{1-\cos\theta}=\frac{2\times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=2\cot\frac{\theta}{2}$$

(20)

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(3\pi-\frac{\pi}{4}+\alpha\right)=$$

$$-\cos\left(\alpha-\frac{\pi}{4}\right)=-\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right)$$

$$=-\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)$$



$k^2=98 \quad k=\sqrt{98}$

$$\cos\alpha=-\frac{\sqrt{98}}{10}=\frac{-7\sqrt{2}}{10}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=-\frac{\sqrt{2}}{2}\left(-\frac{7\sqrt{2}}{10}+\frac{\sqrt{2}}{10}\right)$$

$$\Rightarrow -\frac{\sqrt{2}}{2}\left(-\frac{6\sqrt{2}}{10}\right)=\frac{3}{5}$$

5)

$\tan(285^\circ) = \tan(180^\circ + 90^\circ + 15^\circ) = \tan(90^\circ + 15^\circ) = -\cot 15^\circ$

$\tan(-165^\circ) = -\tan(180 - 15^\circ) = -(-\tan 15^\circ) = \tan 15^\circ$

$\sin(1095) = \sin(3 \times 360 + 15) = \sin 15^\circ$

$\sin(255^\circ) = \sin(180^\circ + 90^\circ - 15^\circ) = -\cos(90^\circ - 15^\circ) = -\sin 15^\circ$

حاصل‌عبارت $\Rightarrow (-\cot 15^\circ)\tan 15^\circ - \sin 15^\circ(-\sin 15^\circ) = -1 + \sin^2 15^\circ$

$= -(1 - \sin^2 15^\circ) = -\cos^2 15$

تاریخ:..................

موضوع:..................

4)

$\sqrt{3}\cos(210^\circ) = \sqrt{3}\cos(270 - 60^\circ) = -\sqrt{3}\sin(60^\circ)$

$\sin(243^\circ) = \sin(270 - 27^\circ) = -\cos(27^\circ)$

$\sqrt{2}\sin 135 \Rightarrow \sqrt{2}\sin(90 + 45) = -\sqrt{2}\cos(45^\circ)$

$\cos(153^\circ) = \cos(180^\circ - 27^\circ) = -\cos(27^\circ)$

$-3 \times \frac{\sqrt{3}}{2} \times -\cos 27 - (-\sqrt{2} \times \frac{\sqrt{2}}{2} \times -\cos 27^\circ) =$

$\frac{3}{2}\cos 27^\circ - \cos 27^\circ = \frac{1}{2}\cos(27^\circ)$

$\frac{\frac{1}{2}\cos 27^\circ}{\cos 27^\circ} = \frac{1}{2}$

(۳)

$\tan x + \cot x = -3 \qquad \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \Rightarrow$

$\frac{1}{\sin x \cos x} = -3 \rightarrow \sin x \cos x = -\frac{1}{3}$

$\frac{1}{\cos^3 x + \sin^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)} =$

$\frac{1}{(\sin x + \cos x)(1 - \sin x \cos x)}$

$A^2 = (\sin x + \cos x)^2 \Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x \Rightarrow$

$1 + 2\sin x \cos x = 1 + 2(-\frac{1}{3}) = \frac{1}{3}$

$A^2 = \frac{1}{3} \rightarrow A = \pm\frac{1}{\sqrt{3}} \qquad 3\pi < 4x < 4\pi$

$\downarrow \frac{3}{4}\pi < x < \pi$

طبق دایره مثلثاتی سوم در (sin x) و cos x دو حالت $\frac{3\pi}{4} < x < \pi$ قرار دارد

$|\cos x| > |\sin x|$ در نتیجه مخرج A باید منفی باشه.

$\frac{1}{\cos^3 x + \sin^3 x} = \frac{1}{A(1-(-\frac{1}{3}))} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = \frac{1}{-\frac{4}{3\sqrt{3}}} = \frac{-3\sqrt{3}}{4}$

(۵

$\tan(285^\circ) = \tan(180^\circ + 90^\circ + 15^\circ) = \tan(90^\circ + 15^\circ) = -\cot 15^\circ$

$\tan(-165^\circ) = -\tan(180 - 15^\circ) = -(-\tan 15^\circ) = \tan 15^\circ$

$\sin(1095) = \sin(3 \times 360 + 15) = \sin 15^\circ$

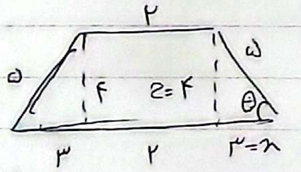
$\sin(255^\circ) = \sin(180^\circ + 90^\circ - 15^\circ) = -\cos(90^\circ - 15^\circ) = -\sin 15^\circ$

جایگذاری $\Rightarrow (-\cot 15^\circ)\tan 15^\circ - \sin 15^\circ(-\sin 15^\circ) = -1 + \sin^2 15^\circ$

$= -(1 - \sin^2 15^\circ) = -\cos^2 15$

(۴

$S = \frac{(\text{قاعده کوچک} + \text{قاعده بزرگ}) \times \text{ارتفاع}}{2}$



ضلع مجاور

$\cos\theta = \frac{6}{10} = \frac{x}{5} \rightarrow x = 3$

وتر

طبق شکل اگه نسبت ۶ به ۱۰ برابر x به ۵ باشه بنابراین x = ۳

طبق فیثاغورث $(z)^2 + (3^2) = (5)^2 \rightarrow z = 4$

جایگذاری $4 \times \frac{2+8}{2} = 20$