نام و نام خانوادگی ...نازنین مرتضوی... پاسخنامه تشریحی تکلیف شماره ...۲۸... کلاس ...۱۱۸...

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$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \to \cancel{\frac{1}{|\cos\alpha|}} - \underbrace{\tan\alpha}_{\frac{\sin\alpha}{\cos\alpha}} = \cancel{\frac{1}{|\cos\alpha|}} + \frac{\sin\alpha}{|\cos\alpha|}$$

$$\frac{-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow |\cos\alpha| = -\cos\alpha \;,\; \Rightarrow \cos\alpha < 0$$

$\sin\alpha$ و $\cos\alpha$ هر دو منفی پس انتهای کمان α در ناحیه سوم است.

$$\frac{|\sin\alpha|}{\cos\alpha} = \frac{1}{-\frac{\cos\alpha}{\sin\alpha}} \Rightarrow \frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow |\sin\alpha| = -\sin\alpha \Rightarrow \sin\alpha < 0$$

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$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \xrightarrow{\times 2} -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \;,\; \sin 2x = \frac{m-1}{4}$$

$$\sin\frac{-\pi}{6} < \sin 2x < \sin\frac{5\pi}{6} \to -\frac{1}{2} < \sin 2x \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \xrightarrow{\times 4} -2 < m-1 \leq 4 \Rightarrow \boxed{-1 < m \leq 5}$$

در بازه $-\frac{\pi}{6}$ تا $\frac{5\pi}{6}$ بیشترین سینوس $\sin\frac{\pi}{2}$ است که یک می‌شود.

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$$\tan x + \cot x = -3 \to \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\cos^2 x + \sin^2 x}{\sin x\cos x} = \frac{1}{\frac{1}{2}\sin 2x} = \frac{2}{\sin 2x} = -3 \;,\; \sin 2x = -\frac{2}{3} \xrightarrow{+1}$$

$$1 + \sin 2x = \frac{1}{3} \to (\sin x + \cos x)^2 = \frac{1}{3} \;,\; \sin x + \cos x = -\frac{1}{\sqrt{3}}$$

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{\underbrace{(\sin x + \cos x)}_{-\frac{1}{\sqrt{3}}}(1 - \underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x,\ -\frac{2}{3}})} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = \boxed{\frac{-3\sqrt{3}}{4}}$$

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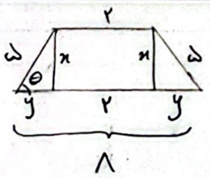

$$\cos\theta = \frac{6}{10} \Rightarrow \sin^2\theta = 1 - \left(\frac{6}{10}\right)^2 = \frac{64}{100} \Rightarrow \sin\theta = 0/8$$

$$\frac{x}{5} = \frac{8}{10} \Rightarrow x = 4 \;,\; \frac{y}{5} = \frac{6}{10} \Rightarrow y = 3$$

S ذوزنقه $= \frac{(2+8)\times 4}{2} = 20$

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$$\tan(\underbrace{285^\circ}_{\substack{270+15\\ \text{ربع ۴}}})\tan(\underbrace{-165^\circ}_{\substack{-(180-15)\\ \text{ربع ۲}}}) - \sin(\underbrace{1095}_{\substack{(1080+15)\\ \text{ربع ۱}}})\cos(\underbrace{255^\circ}_{\substack{270-15\\ \text{ربع ۳}}}) = K\cos^2 15$$

$$(-\cot 15^\circ)(\tan 15^\circ) - (\sin 15^\circ)(-\sin 15^\circ) = -1 + \sin^2 15 = -\cos^2 15 \Rightarrow K = -1$$

$A = \sqrt{3}\cos(\underbrace{210^\circ}_{180+30})\sin(\underbrace{243^\circ}_{180+63}) - \sqrt{2}\sin(\underbrace{135^\circ}_{180-45})\cos(\underbrace{153^\circ}_{180-27})$ $A = \sqrt{3}(-\cos 30^\circ)\times(-\sin 63) - \sqrt{2}(\sin 45)\times(-\cos 27^\circ) = \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)\times -\sin 63 - \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)\times -\cos 27$ $\left.\begin{array}{l}\frac{3}{2}\sin 63 + \cos 27 \\ 27 + 63 = 90 \Rightarrow \sin 63 = \cos 27\end{array}\right\\} \Rightarrow \frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27 \Rightarrow \boxed{\frac{\frac{5}{2}\cos 27}{\cos 27} = \frac{5}{2}}$	۶
$f(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x) \rightarrow f\left(\frac{\pi}{36}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}\cos^2\frac{2\pi}{3}$ $\cos^2\left(\frac{\pi}{12}\right) = \frac{1+\cos\frac{\pi}{6}}{2} = 1 + \frac{\sqrt{3}}{2}$ $\Rightarrow 16\left(1+\frac{\sqrt{3}}{2}\right)\times\left(\frac{\sqrt{3}}{2}\right)^2\times\left(\frac{1}{2}\right)^2\times\left(-\frac{1}{2}\right)^2 = \frac{6+3\sqrt{3}}{16}$	۷
$\sin x, \cos x < 0$ ، $\frac{1-\sin x}{1+\sin x} = 4 \rightarrow 4 + 4\sin x = 1 - \sin x$ ، $5\sin x = -3$, $\sin x = \frac{-3}{5}$ $\sin 2x = \frac{2\tan x}{1+\tan^2 x} \rightarrow \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{-3}{5} \Rightarrow 10\tan\frac{x}{2} = -3 - 3\tan^2\frac{x}{2}$ $3t^2 + 10t + 3 = 0 \quad \Delta = 64 \rightarrow t = \frac{-10\pm 8}{6} = -3, \frac{-1}{3} \Rightarrow \tan\frac{x}{2} = -\frac{1}{3}, -3$	۸
$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = K\cot\frac{\theta}{2}$ ، $1-\cos 2\theta = 2\sin^2\theta$ ، $1+\cos 2\theta = 2\cos^2\theta$ $\Rightarrow \frac{\sin\theta}{2\sin^2\frac{\theta}{2}} + \frac{2\cos^2\frac{\theta}{2}}{\sin\theta}$ ، $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \Rightarrow \frac{\cancel{2\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}{\cancel{2\sin^2\frac{\theta}{2}}} + \frac{2\cos^{\cancel{2}}\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = 2\cot\frac{\theta}{2} \Rightarrow K = 2$	۹
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{2}{100} \Rightarrow \cos\alpha = \frac{-\sqrt{98}}{10}$ ربع دوم $\Leftarrow \cos\alpha < 0$ $\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\frac{11\pi}{4}\times\cos\alpha - \sin\frac{11\pi}{4}\times\sin\alpha$ ، $\frac{11\pi}{4} = 2\pi + \frac{3\pi}{4} \Rightarrow \cos\frac{11\pi}{4} = -\frac{\sqrt{2}}{2}$, $\sin\frac{11\pi}{4} = \frac{\sqrt{2}}{2}$ $\Rightarrow \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{98}}{10}\right) - \left(\frac{\sqrt{2}}{2}\right)\times\left(\frac{\sqrt{2}}{10}\right) = \frac{14}{20} - \frac{2}{20} = \frac{3}{5}$	۱۰