۲۷

سانیا دمادیان

① $S_\Delta = \frac{۱}{۲} AB \sin\alpha \longrightarrow ۲/۵ = \frac{۶\sqrt{۳}}{۲} \times \sin\alpha$

$\sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \boxed{\begin{matrix} max = ۱۲۰ \\ min = ۶۰ \end{matrix}}$

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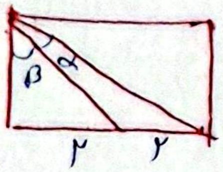
② $\tan B = \frac{۲}{۲} = ۱ \qquad \alpha + B = \theta \rightarrow \tan\theta = ۶/۲ = ۳$

$\tan\alpha = \tan(\theta - B) = \frac{\tan\theta - \tan B}{۱ + \tan\theta \tan B} \Rightarrow \frac{۳ - ۱}{۱ + ۳} = \frac{۱}{۲} \Rightarrow \boxed{\cot\alpha = ۲}$



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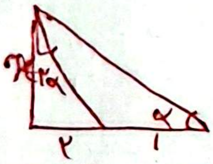
③ $\cot ۲\alpha = x/۲ \ , \ \cot\alpha = ۳/x \ , \ \cot ۲\alpha = \frac{\cot^۲\alpha - ۱}{۲\cot\alpha}$

$\Rightarrow \frac{x}{۲} = \frac{\frac{۹ - x^۲}{x^۲}}{\frac{۶}{x}} \Rightarrow \frac{x}{۲} = \frac{۹ - x^۲}{۶x} \Rightarrow ۳x^۲ = ۹ - x^۲ \Rightarrow x^۲ = \frac{۹}{۴} \Rightarrow x = \frac{۳}{۲}$

$\boxed{\cot\alpha = ۲}$



$\frac{۳}{\frac{۳}{۲}} = ②$

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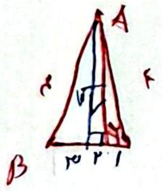
④ $h = \sqrt{۷} \rightarrow \tan(۱۸۰ - \alpha) = -\tan\alpha \Rightarrow -\tan\alpha = \frac{\sqrt{۷}}{۳}$

$\rightarrow \tan\alpha = \boxed{-\frac{\sqrt{۷}}{۳}}$



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⑤ $۲\sin^۲x + \cos^۲x = ۴/۳ \longrightarrow \overset{۱}{\cancel{\sin^۲x + \cos^۲x}} + \sin^۲x = \frac{۴}{۳}$

$\Rightarrow \sin^۲x = ۱/۳ \rightarrow \cot^۲x + ۱ = \frac{۱}{\sin^۲x} \rightarrow \cot^۲x = ۲ \rightarrow \boxed{\tan^۲x = \frac{۱}{۲}}$

(۶)

$$\frac{\sin^۴\alpha+۴\cos^۲\alpha}{۱+\cos^۲\alpha}-\frac{\cos^۴\alpha+۴\sin^۲\alpha}{۱+\sin^۲\alpha}\Rightarrow\frac{\cos^۴\alpha+۲\cos^۲\alpha+۱}{۱+\cos^۲\alpha}-\frac{\sin^۴\alpha+۲\sin^۲\alpha+۱}{۱+\sin^۲\alpha}$$

$$\Rightarrow\frac{(۱+\cos^۲\alpha)^۲}{۱+\cos^۲\alpha}-\frac{(\sin^۲\alpha+۱)^۲}{\sin^۲\alpha+۱}=۱+\cos^۲\alpha-۱-\sin^۲\alpha=\boxed{\cos ۲\alpha}$$

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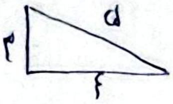
(۷)

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right)-\tan\left(\alpha+\frac{۳\pi}{۲}\right)=$$

$\cos\alpha \cdot -\sin\alpha+\cot\alpha$



$$\Rightarrow\frac{۳}{۵}\times-\frac{۴}{۵}+\frac{۳}{۴}=\boxed{۰/۲۷}$$

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(۸)

$$(۳\cos ۲\alpha+\sqrt{۲}\sin x-\sqrt{۲}\cos x)=۳\cos\frac{\pi}{۳}-\sqrt{۲}(\cos x-\sin x)$$

$$\frac{۳}{۲}-\sqrt{۲}\left(\frac{\cos^۲x-\sin^۲x}{\cos x+\sin x}\right)=\frac{۳}{۲}-\sqrt{۲}\left(\frac{\cos\frac{\pi}{۶}}{\sqrt{۲}\sin(\frac{\pi}{۱۲}+\frac{\pi}{۴})}\right)\Rightarrow\frac{۳}{۲}-\frac{\frac{\sqrt{۳}}{۲}}{\frac{\sqrt{۳}}{۲}}=\boxed{\frac{۱}{۲}}$$

$\sin(\frac{\pi}{۱۲}+\frac{\pi}{۴}) \rightarrow \sin\frac{\pi}{۳}$

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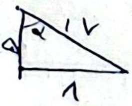
(۹)

$$\tan\alpha=\frac{۲\tan(\frac{\alpha}{۲})}{۱-(\tan(\frac{\alpha}{۲}))^۲}\Rightarrow\tan\alpha=\frac{\frac{۱}{۲}}{\frac{۱۵}{۱۶}}=\frac{۸}{۱۵}\rightarrow$$



$$\Rightarrow\frac{\frac{۸}{۱۵}-\frac{۸}{۱۷}}{\frac{۸}{۱۷}-\frac{۱۵}{۱۷}}=\frac{\frac{۸(۱۷-۱۵)}{۱۵\times۱۷}}{\frac{۸-۱۵}{۱۷}}=\boxed{\frac{۱۶}{۱۰۵}}$$

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(۱۰)

$\frac{\cot\alpha}{\sin\alpha}>۰ \longrightarrow \frac{\cos\alpha}{\sin^۲\alpha}>۰ \rightarrow \cos\alpha>۰ \rightarrow$ اول یا چهارم

$۲\sin\alpha<۲\sin\alpha\cos\alpha$
- $\rightarrow \sin\alpha>۰ \longrightarrow \cos\alpha>۰$
- $\rightarrow \sin\alpha<۰ \longrightarrow \cos\alpha<۰$

اول یا سوم

$\boxed{\text{چهارم}}$