سایا دادیان

۱۹٫۵

۱

$S_\Delta = \frac{۱}{۲} AB \sin\alpha \longrightarrow ۲/۵ = \frac{۶\sqrt{۳}}{۲} \times \sin\alpha$

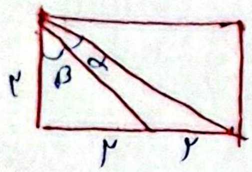
$\sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \boxed{\begin{array}{l} max = ۱۲۰ \\ min = ۶۰ \end{array}}$ $\frac{max}{min} = ۲$

۱٫۷۵

۲

$\tan B = \frac{۲}{۲} = ۱$ $\quad \alpha + B = \theta \rightarrow \tan\theta = \frac{۶}{۲} = ۳$

$\tan\alpha = \tan(\theta - B) = \frac{\tan\theta - \tan B}{۱ + \tan\theta \tan B} \Rightarrow \frac{۳-۱}{۱+۳} = \frac{۱}{۲} \Rightarrow \boxed{\cot\alpha = ۲}$

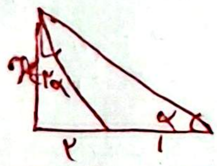


۳

$\cot ۲\alpha = \frac{x}{۲}$, $\cot\alpha = \frac{۳}{x}$, $\cot ۲\alpha = \frac{\cot^۲\alpha - ۱}{۲\cot\alpha}$

$\Rightarrow \frac{x}{۲} = \frac{\frac{۹-x^۲}{x^۲}}{\frac{۶}{x}} \Rightarrow \frac{x}{۲} = \frac{۹-x^۲}{۶x} \Rightarrow ۳x^۲ = ۹ - x^۲ \Rightarrow x^۲ = \frac{۹}{۴} \Rightarrow x = \frac{۳}{۲}$

$\boxed{\cot\alpha = ۲}$

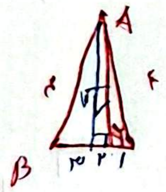


$\frac{\frac{۳}{۱}}{\frac{۳}{۲}} = ۲$

۴

$h_a = \sqrt{۷} \rightarrow \tan(۱۸۰ - \alpha) = -\tan\alpha \Rightarrow -\tan\alpha = \frac{\sqrt{۷}}{۳}$

$\rightarrow \tan\alpha = \boxed{-\frac{\sqrt{۷}}{۳}}$



۵

$۲\sin^۲x + \cos^۲x = \frac{۴}{۳} \longrightarrow \overbrace{\sin^۲x + \cos^۲x}^{۱} + \sin^۲x = \frac{۴}{۳}$

$\Rightarrow \sin^۲x = \frac{۱}{۳} \rightarrow \cot^۲x + ۱ = \frac{۱}{\sin^۲x} \rightarrow \cot^۲x = ۲ \rightarrow \boxed{\tan^۲x = \frac{۱}{۲}}$

۶

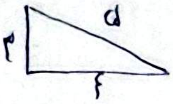
$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}\Rightarrow\frac{\cos^4\alpha+2\cos^2\alpha+1}{1+\cos^2\alpha}-\frac{\sin^4\alpha+2\sin^2\alpha+1}{1+\sin^2\alpha}$$

$$\Rightarrow\frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha}-\frac{(\sin^2\alpha+1)^2}{\sin^2\alpha+1}=1+\cos^2\alpha-1-\sin^2\alpha=\boxed{\cos 2\alpha}$$

۷

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha+\frac{3\pi}{2}\right)=$$

$$\cos\alpha\cdot-\sin\alpha+\cot\alpha$$



$$\Rightarrow\frac{3}{5}\times-\frac{4}{5}+\frac{3}{4}=\boxed{0/27}$$

۸

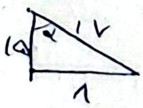
$$(3\cos 2x+\sqrt{2}\sin x-\sqrt{2}\cos x)=3\cos\frac{\pi}{3}-\sqrt{2}(\cos x-\sin x)$$

$$-\frac{3}{2}-\sqrt{2}\left(\frac{\cos^2 x-\sin^2 x}{\cos x+\sin x}\right)=\frac{3}{2}-\sqrt{2}\left(\frac{\cos\frac{\pi}{6}}{\sqrt{2}\sin\left(\frac{\pi}{12}+\frac{\pi}{4}\right)}\right)\Rightarrow\frac{3}{2}-\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}=\boxed{\frac{1}{2}}$$

$$\sin\left(\frac{\pi}{12}+\frac{\pi}{4}\right)\to\sin\frac{\pi}{3}$$

۹

$$\tan\alpha=\frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\left(\tan\left(\frac{\alpha}{2}\right)\right)^2}\Rightarrow\tan\alpha=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}\to$$



$$\Rightarrow\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{8(17-15)}{\frac{15\times17}{\frac{8-15}{17}}}=\boxed{-\frac{16}{105}}$$

۱/۷۵

۱۰

$$\frac{\cot\alpha}{\sin\alpha}>0\to\frac{\cos\alpha}{\sin^2\alpha}>0\to\cos\alpha>0\to$$ اول یا چهارم

$$2\sin\alpha<2\sin\alpha\cos\alpha$$
$\to \sin\alpha>0\to\cos\alpha>1$
$\to \sin\alpha<0\to\cos\alpha<1$ سوم یا چهارم

$\boxed{\text{چهارم}}$